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BET-021

DIPLOMA IN CIVIL ENGINEERING (DCLE(G)) / DIPLOMA IN MECHANICAL ENGINEERING (DME) / DCLEVI / DMEVI / DELVI / DECVI / DCSVI/ ACCLEVI / ACMEVI / ACELVI / ACECVI / ACCSVI

Term-End Examination

00661

December, 2019

BET-021 : MATHEMATICS – II

Time : 2 hours *Maximum Marks : 70*

Note : *Question no. **1** is **compulsory**. Attempt any **four** questions out of the remaining. Use of scientific calculator is permitted.*

1. Answer any ***seven*** from the following : *7×2=14*

(a) Examine the following function as even or odd :

$$f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x + x^2}$$

(b) Find the value of

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x}{1 - \sin x}.$$

(c) Examine the continuity of f(x) at x = 0 where,

$$f(x) = \begin{cases} \dfrac{|\sin x|}{x} & \text{when} \quad x \neq 0 \\ 1 & \text{when} \quad x = 0. \end{cases}$$

(d) Find $\dfrac{dy}{dx}$:

$x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$

(e) Show that $(x^2 - 3x^2 + 4x)$ will increase with the increase of x.

(f) Find the value of

$$\int \sqrt{1 + \sin\frac{x}{2}}\, dx .$$

(g) Find the value of

$$\int_0^{\pi/2} x^2 \sin x\, dx.$$

(h) If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then find the values of x and y.

(i) If $P = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$, then show that $P^2 = P$.

(j) The average of five numbers is 12; if four numbers out of them are 8, 11, 13 and 17, then find the fifth number.

2. (a) Evaluate :

$$\lim_{x \to 2} \frac{x^6 - 24x - 16}{x^3 + 2x - 12}$$

(b) A closed circular cylinder has height 16 cm and radius r cm. The total surface area is A cm^2. Prove that : $\frac{dA}{dt} = 4\pi(r + 8)\frac{dr}{dt}$. Hence calculate the rate of increase in area, if the radius increases at the rate of 0·02 cm, when radius is 4 cm. *2×7=14*

3. (a) If p, q be the imaginary cube roots of unity, prove that $p^2 + q^2 - pq = -2$.

(b) ω is an imaginary cube root of unity and $x = a + b$, $y = a\omega + b\omega^2$, $z = a\omega^2 + b\omega$; show that $xyz = a^3 + b^3$. *2×7=14*

4. (a) Find the equations of the tangents to the ellipse $2x^2 + 3y^2 = 30$ which are parallel to the straight line $x + y + 18 = 0$.

(b) Integrate : *2×7=14*

$$\int \left(x^3 + 6x^2 + 9x + \frac{6}{x}\right) dx$$

5. (a) Find the area included between $y^2 = 9x$ and $y = x$.

(b) Find the maximum value of

$$4 \sin x + 3 \cos x.$$

2×7=14

6. (a) Prove that :

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

(b) Solve the following equations by matrix method :

$$x + y + z - 7 = 0$$

$$x + 2y + 3z - 16 = 0$$

$$x + 3y + 4z - 22 = 0$$

2×7=14

7. (a) Find the mean and median from the following :

Marks	*No. of students*
Under 10	175
Under 20	360
Under 30	680
Under 40	790
Under 50	900
Under 60	1000

(b) The following is the frequency table showing the heights of 200 boys :

Heights (in inches)	*No. of students*
53	7
55	14
57	31
59	60
61	52
63	29
65	4
67	3

Calculate the standard deviation. *2×7=14*