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BME-001

B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination

December, 2019

00653

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of calculator is allowed.*

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1. Attempt any ***five*** of the following : *5×4=20*

(a) Discuss the limit of the function f defined below :

$$f(x) = \begin{cases} \frac{1}{2} - x & \text{if } 0 < x < \frac{1}{2} \\ \frac{3}{2} - x & \text{if } \frac{1}{2} < x < 1 \end{cases}$$

at $x = \frac{1}{2}$

(b) Show that the function

$$f(x) = \begin{cases} \dfrac{e^{1/x} - 1}{e^{1/x} + 1}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

is not differentiable at $x = 0$.

(c) Give the Maclaurin series expansion of function sin x.

(d) If $y = \sin [m (\sin^{-1} x)]$, prove that

$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} + (m^2 - n^2) y_n = 0.$$

(e) Solve any ***one*** of the following differential equations :

(i) $(x^2 - y^2) \dfrac{dy}{dx} = xy$

(ii) $\dfrac{dy}{dx} = e^{x - y} + x^2 e^{-y}$

(f) Evaluate any ***one*** of the following :

(i) $\displaystyle\int_1^2 x^2 \log x \, dx$

(ii) $\displaystyle\int_0^{\pi/2} \frac{\sin x}{(1 + \cos x)^2} dx$

2. Answer any ***four*** of the following : *4×5=20*

(a) If $\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$, find $\vec{A} \times \vec{B}$ and the unit vector perpendicular to both $\vec{A}$ and $\vec{B}$. Also find the sine of the angle between $\vec{A}$ and $\vec{B}$.

(b) A particle is acted on by constant forces $2\hat{i} + \hat{j} - \hat{k}$, $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} + \hat{j} + 5\hat{k}$ and is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $6\hat{i} + 3\hat{j} + \hat{k}$. Find the work done.

(c) Define a solenoidal vector function. Show that the vector function $\vec{F}$ given by

$$\vec{F} = (y - z)\,\hat{i} + (z - x)\,\hat{j} + (x - y)\,\hat{k}$$

is a solenoidal function. Find the vector function $\vec{f}$ such that curl $\vec{f} = \vec{F}$.

(d) If $\vec{OA} = a\hat{i}, \vec{OB} = a\hat{j}$ and $\vec{OC} = a\hat{k}$, form three co-terminous edges of a cube and S denotes the surface of the cube, evaluate

$$\int_S \left[(x^2 - yz)\,\hat{i} - 2x^2y\,\hat{j} + 2\hat{k}\right] \cdot \hat{n}\, dS$$

by expressing it as a volume integral.

(e) If $\vec{a}$ is a constant vector and

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k},$$

show that $\text{curl}\,(\vec{a} \times \vec{r}) = 2\vec{a}$.

(f) Charges $e_1, e_2, \ldots, e_n$ are placed at the points $A_1, A_2, \ldots, A_n$. Find the intensity $\vec{E}$ at any point of the field. Hence show that the direction of the line of forces at any point P (where a unit charge is placed) is obtained by joining the centroid G to P.

(g) Find the directional derivative of

$$(x^2 + y^2 + 2xyz) \text{ at } (-1, 2, -2)$$

in the direction of $(3\hat{i} - 3\hat{j} + \hat{k})$.

3. Answer any ***five*** of the following : *5×3=15*

(a) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$,

obtain the product AB. Explain why BA is not defined.

(f) 12 dice were thrown 4096 times and a throw of 6 was reckoned as a success. The observed frequencies were as given below :

Number of successes	Frequencies
0	447
1	1145
2	1181
3	796
4	380
5	115
6	24
7	8
Total	4096

Find the value of χ^2 on the hypothesis that the dice were unbiased and hence show that the data is consistent with the hypothesis so far as χ^2 test is concerned. (Given that $\chi^2_{0.05}$ (for n = 7) = 14·07)

(c) In a certain factory producing tyres, there is a small chance of 1 in 500 tyres to be defective. The tyres are supplied in lots of 10.

Using Poisson distribution, calculate the approximate number of lots containing no defective, 1 defective and 2 defective tyres, respectively, in a consignment of 10,000 lots.

(d) If the probability that a person suffers from cancer is 0·001, determine the probability that out of 2000 persons, the number of persons suffering from cancer are

(i) exactly 3 individuals,

(ii) more than 2 individuals,

(iii) none, and

(iv) more than one individual.

(e) A manufacturer intends that his electric bulbs have a life of 1000 hours. He tests a sample of 20 bulbs, drawn at random from a batch and finds that the mean life of the sample bulbs is 900 hours with a standard deviation of 22 hours. Does this signify that the batch is not up to the standard ? (Given that $t_{0 \cdot 025} = 2 \cdot 093$ for 19 degrees of freedom)

(f) Find K, L and M so that the matrix

$$\begin{bmatrix} i & 7-4i & L \\ K & 2i & 3+i \\ -2-5i & M & -3i \end{bmatrix}$$

is a Skew-Hermitian matrix.

(g) Find the eigenvector of the matrix

$$A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}.$$

4. Answer any ***three*** of the following : *3×5=15*

(a) In a relay race, there are five teams : A, B, C, D and E.

(i) What is the probability that A, B and C finish first, second and third, respectively ?

(ii) What is the probability that A, B and C are first three to finish (in any order) ?

(Assume that all finishing orders are equally likely).

(b) A random variable x has the following probability function :

X	– 2	– 1	0	1	2	3
P(X)	0·1	K	0·2	2K	0·3	K

Find the value of K and calculate the mean and variance.

(b) Determine the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}.$$

(c) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 7 & 10 \\ 3 & 6 & 10 & 12 \end{bmatrix}.$$

(d) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that $A^2 - 4A + 5I = O$, where I and O are respectively the unit matrix and null matrix of order 3. Use this result to find A^{-1}.

(e) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$