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**MSTE-002**

# POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

## Term-End Examination

## December, 2018

00132

## MSTE-002 : INDUSTRIAL STATISTICS-II

*Time : 3 hours* *Maximum Marks : 50*

***Note :***

(i) *Attempt **all** questions. Questions no. 2 to 5 have internal choices.*

(ii) *Use of scientific calculator is allowed.*

(iii) *Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.*

(iv) *Symbols have their usual meanings.*

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1. State whether the following statements are *True* or *False*. Give reasons in support of your answers. *5×2=10*

   (a) A set R is convex if and only if $(x_1, y_1) \in R$ and $(x_2, y_2) \in R \Rightarrow$
   $(\lambda x_1 + (1 - \lambda)x_2, (1 - \lambda) y_1 + \lambda y_2) \in R, 0 \leq \lambda \leq 1.$

   (b) In stepwise selection method, a regressor variable once selected in the regression model, can never be deleted from the model.

(c) There is a transportation problem having 4 origins and 5 destinations. To apply optimality test to a feasible solution, there must be 9 independent allocations.

(d) Assume that customers are arriving randomly at a coffee shop. If $X_n$ denotes number of customers at the shop at the end of $n^{th}$ hour, then the random process $\{X_n : n \in T\}$ is a continuous parameter process.

(e) The residual is the absolute difference between the observed value of the response variable and the predicted value of the response variable.

**2.** (a) Show that the set $S = \{(x, y) : x^2 + y^2 \leq 4\}$ is a convex set. *5*

(b) Solve the following LPP by graphical method :

Minimise $z = 3x + 2y$

subject to :

$$2x + y \leq 18$$

$$2x + 3y \leq 42$$

$$3x + y \leq 24$$

$$x \geq 0, y \geq 0$$ *5*

**OR**

Using simplex method, solve the following LPP :

Minimise $z = 10x_1 + 4x_2$

subject to the constraints :

$$-x_1 + x_2 \leq 2$$
$$x_1 + x_2 \geq 4$$
$$x_1, x_2 \geq 0$$

*10*

3. (a) A company is producing a single product and selling it through five agencies situated in different cities. All of a sudden, there is a demand for the product in five more cities that do not have any agency of the company. The company is faced with the problem of deciding on how to assign the existing agencies to dispatch the product to the additional cities in such a way that the travelling distance is minimised. The distances (in km) between the surplus and deficit cities are given in the following distance matrix :

| Surplus city \ Deficit city | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 160 | 130 | 175 | 190 | 200 |
| B | 135 | 120 | 130 | 160 | 175 |
| C | 140 | 110 | 155 | 170 | 185 |
| D | 50 | 50 | 80 | 80 | 110 |
| E | 55 | 35 | 70 | 80 | 105 |

Determine the optimum assignment schedule. *8*

(b) Customers arrive at a window in a bank according to a Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of the window including that for the serviced customers can accommodate a maximum of three customers. Other customers can wait outside this space. What is the probability that an arriving customer can go directly to the space in front of the window ? 2

**OR**

(a) There are 5 jobs, each of which has to go through the machines A and B in the order AB. The processing times (in hours) are given as follows :

| Job | $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ |
|---|---|---|---|---|---|
| Machine A | 2 | 4 | 5 | 7 | 1 |
| Machine B | 3 | 6 | 1 | 4 | 8 |

Determine a sequence of these jobs that will minimise the total elapsed time T. Also obtain :

(i) Minimum elapsed time.

(ii) Idle time for each machine. 5

(b) A shop has five machines A, B, C, D, E. Two jobs must be processed through each of these machines. The time (in hours) taken on each of these machines and the necessary sequence of jobs through the machines are given below :

| | | | | | | |
|---|---|---|---|---|---|---|
| Job 1 | Sequence | A | B | C | D | E |
| | Time | 2 | 4 | 3 | 6 | 6 |
| Job 2 | Sequence | C | A | D | E | B |
| | Time | 4 | 6 | 3 | 3 | 6 |

Use graphical method to obtain the total minimum elapsed time. 5

4. A statistician collected data of 25 values for two independent random variables $X_1$ and $X_2$. The following four models were fitted :

$$Y = B_0 + e$$

$$Y = B_0 + B_1 X_1 + e$$

$$Y = B_0 + B_2 X_2 + e$$

$$Y = B_0 + B_1 X_1 + B_2 X_2 + e$$

The results obtained were :

$\hat{B}_0 = 22{\cdot}38$, $\hat{B}_1 = 1{\cdot}6161$, $\hat{B}_2 = 0{\cdot}0144$,

$SS(B_0) = 12526{\cdot}08$, $SS(B_0, B_1) = 17908{\cdot}47$,

$SS(B_0, B_2) = 17125{\cdot}23$, $SS(B_0, B_1, B_2) = 18079{\cdot}0$,

$\hat{\sigma}^2 = 10{\cdot}53$, $SE(\hat{B}_1) = 0{\cdot}17$ and $SE(\hat{B}_2) = 0{\cdot}0035$.

(a) Find the additional contribution of (i) $X_2$ over $X_1$, and (ii) $X_1$ over $X_2$. 4

(b) Test whether their inclusion in the model is justified or not. 6

**OR**

A statistician is analysing the vending machine routes in the distribution system for predicting the amount of time required by the route driver to service the vending machines in an outlet. The company manager has suggested that the two most important variables affecting the delivery time Y (in minutes), are :

(a) number of cases ($X_1$), and

(b) distance travelled (in metres) by the route driver ($X_2$). The delivery time data are given below :

| Time (Y) | No. of cases ($X_1$) | Distance ($X_2$) |
|---|---|---|
| 20 | 10 | 50 |
| 10 | 5 | 20 |
| 10 | 5 | 30 |
| 15 | 5 | 10 |
| 15 | 10 | 10 |
| 20 | 10 | 30 |
| 10 | 5 | 10 |
| 25 | 15 | 40 |
| 30 | 10 | 80 |
| 15 | 10 | 20 |
| 20 | 10 | 10 |
| 10 | 5 | 40 |

(i) Fit multiple regression model. Also predict the expected time at $X_1 = 7$ and $X_2 = 20$. *7*

(ii) Estimate the variance of error terms $\sigma^2$. *3*

**5.** (a) Find the pacf of AR (2) process : *5*

$$X_t = 0{\cdot}333\, X_{t-1} + 0{\cdot}222\, X_{t-2} + a_t$$

(b) For the model

$$(1 - 0{\cdot}2\, B)\,(1 - B)X_t = (1 - 0{\cdot}5\, B)\, a_t,$$

find p, d, q and express it as ARIMA (p, d, q). Determine whether the process is stationary and invertible. *5*

**OR**

(a) Define Autoregressive process. *2*

(b) For the following autoregressive model :

$$X_t = 0{\cdot}7\, X_{t-1} - 0{\cdot}4\, X_{t-2} + a_t,$$

(i) Verify whether the series is stationary.

(ii) Obtain $\rho_k$, k = 1, 2, 3, 4 and 5.

(iii) Plot the correlogram. *8*