MST-003

POST GRADUATE DIPLOMA IN
APPLIED STATISTICS (PGDAST)

Term-End Examination

01332

December, 2018

MST-003 : PROBABILITY THEORY

Time : 3 hours   *Maximum Marks : 50*

Note :

   (i)   Attempt **all** questions. Questions no. 2 to 5 have
          internal choices.

   (ii)   Use of scientific calculator is allowed.

   (iii) Use of Formulae and Statistical Tables Booklet for
          PGDAST is allowed.

   (iv)   Symbols have their usual meanings.

1.   State whether the following statements are *True*
     or *False*. Give reasons in support of your
     answers.   5×2=10

     (a) The odds in favour that a person speaks the
          truth are 3 : 2, then the probability that the
          person speaks truth will be 2/3.

(b) If A and B are independent events with $P(A) = 0.2$, $P(B) = 0.5$, then $P(A \cap B) = 0.7$.

(c) If variance of a random variable x is 3, then variance of $y = 2x + 5$ will be 6.

(d) If mean and variance of Negative Binomial distribution are 3 and 5, respectively, then p will be 3/5.

(e) A random variable X follows normal distribution having p.d.f

$$f(x) = \frac{1}{2\sqrt{2\pi}} \ e^{-\frac{1}{8}(x-2)^2} \quad ; \ -\infty < x < \infty$$

The variance of the random variable x is 4.

2. (a) A person is known to hit the target in 3 out of 4 shots whereas another person is known to hit 2 out of 5 shots. Find the probability of the target being hit when both of them are trying.   5

(b) If three dice are thrown simultaneously, then find the probability of getting a

(i) triplet

(ii) sum of 5

(iii) sum of at least 16.   1+2+2

OR

(a)   A card is drawn from a well-shuffled pack of cards. If the drawn card is a face card, what is the probability that it is a king ?   *5*

(b)   In a bolt factory, machines A, B and C manufacture 25%, 35% and 40% of the total product respectively. Out of these 5%, 4% and 2% produced by machines A, B and C, respectively are defective bolts. A bolt is drawn at random from the lot and is found to be defective. What is the probability that it was manufactured by machine B ?   *5*

3.   (a)   A discrete random variable X has the following probability distribution :

X	0	1	2	3	4	5
p(x)	0	C	C	2C	3C	C

Find :

(i)   The constant C

(ii) $P[X < 3]$, and

(iii) $P[1 < X \leq 4]$.   *1+2+2*

(b) Two discrete random variables X and Y have

$$P[X = 0, Y = 0] = \frac{2}{9}, P[X = 0, Y = 1] = \frac{1}{9},$$

$$P[X = 1, Y = 0] = \frac{1}{9} \text{ and } P[X = 1, Y = 1] = \frac{5}{9}$$

Examine whether X and Y are independent. 5

OR

(a) Let X and Y be two random variables. Then for

$$f(x, y) = \begin{cases} kxy & ; \quad 0 < x < 4, 1 < y < 5 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

to be a joint density function, what must be the value of k ? Also find marginal density functions of X and Y. 6

(b) If it rains, a raincoat dealer can earn ₹ 800 per day. If it is a dry day, he can lose ₹ 100 per day. If the probability of rain on any particular day is 0·4, find his expected earning. 4

4. (a) An oil exploration firm plans to drill six holes. It is believed that the probability that each hole will yield oil is 0·2. Since the holes are in quite different locations, the outcome of drilling one hole is statistically independent of that of drilling any of the other holes. What is the probability that two or more holes produce oil ? 5

(b) If a random variable X follows Poisson distribution such that

$$P[X = 1] = 2P[X = 2],$$

find the mean and variance of the distribution and also find $P[X \leq 1]$.　　5

OR

(a) A taxi cab company has 12 Innovas and 8 Maruti Zens. If 5 of these taxi cabs are in the shop for repairs and Innova is as likely to be in for repairs as a Maruti Zen, what is the probability that

　(i) 3 of them are Innovas and 2 are Maruti Zens ?

　(ii) All 5 of them are Maruti Zens ?　　5

(b) If a boy is throwing stones at a target, what is the probability that his 10th throw is his 5th hit, if the probability of hitting the target at any trial is 0·5 ?　　5

5. (a) For a normal distribution, the first moment about 5 is 30 and the fourth moment about 35 is 768. Find the mean and standard deviation of the distribution.　　5

(b) Suppose that temperature of a particular city in the month of March is normally distributed with mean 24°C and standard deviation 6°C. Find the probability that the temperature of the city on a particular day in the month of March is between 23°C and 27°C. 5

OR

(a) The magnitude of earthquakes recorded in a region of a country can be modelled as having an exponential distribution with a mean 2·4 (as measured on the Richter Scale). Find the probability that an earthquake striking this region will

    (i) exceed 3·0 on the Richter Scale.

    (ii) fall between 2·0 and 3·0 on the Richter Scale. 5

(b) The amount of time, say, X (in minutes) that a person has to wait for a bus is uniformly distributed with mean 5 minutes and variance 3 minutes. Find $P(X < 4)$. 5

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