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MMTE-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00022

December, 2018

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Answer any **four** questions from questions no. 1 to 5. Question no. 6 is compulsory. All question carry equal marks. Calculators are not allowed.*

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1. (a) Define the following, giving an example of each : 6

(i) Linear code

(ii) Dual of a code

(iii) Parity check matrix

(b) Compute the 3-cyclotomic cosets modulo 8. 4

2. (a) Construct a parity check matrix of the (7, 4) binary Hamming code. Using this parity check matrix, decode the following vectors, and then check that your decoded vectors are actually codewords :

(i) (0011010),

(ii) (1011110). 7

(b) Consider a binary code with generator matrix

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

Reduce the generator matrix to standard form. Find the parity check matrix of the code using it. *3*

3. (a) Let $\mathcal{C}$ be the (2, 1) convolutional code with generator matrix

$$G = [1 + D + D^2 \quad 1 + D^2].$$

(i) Give the resulting codeword $[C_1 \quad C_2]$ if 110101 is encoded. Also give the interleaved output.

(ii) Give the equations for $C_1(i)$ and $C_2(i)$ for the encoder G, where i = 1, 2. *6*

(b) Define a perfect code. Is the code $\mathcal{C}$ with the following parity check matrix a perfect code ? Give reasons for your answer.

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

How many errors can $\mathcal{C}$ correct ? *4*

4. (a) Write all possible generator polynomials of a (7, 4)-cyclic code. Also find the generator matrix and the parity check matrix of the (7, 4) cyclic code generated by $X^3 + X^2 + 1$ over GF(2). *4*

(b) Construct the generating idempotents of the duadic codes of length 5 over $\mathbf{F}_4$. *6*

5. (a) Show that a BCH code of designed distance δ has minimum weight at least δ. *4*

(b) Let $\mathcal{C}$ be the [5, 2] binary code generated by $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$. Find the weight distribution of $\mathcal{C}$. Find the weight distribution of $\mathcal{C}^{\perp}$ by using the MacWilliams Identity. *6*

6. Which of the following statements are *True* and which are *False* ? Justify your answer with a short proof or a counter example. *10*

(a) The code rate of Hamming code of length

$$2^l - 1 \text{ is } 1 + \frac{l}{2^l - 1}.$$

(b) Every code of even length is self-dual.

(c) $\mathbf{F}_4$ is a subfield of $\mathbf{F}_8$.

(d) There is no quadratic residue code of length 5 over $\mathbf{F}_4$.

(e) Extending a code does not increase the length of the code.