No. of Printed Pages : 3

MMTE-001

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00922

Term-End Examination

December, 2018

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no. 1 is **compulsory**. Answer any **four** questions from questions no. **2** to **6**. Electronic computing devices are not allowed. Draw diagrams wherever necessary.*

1. State whether the following statements are *true* or *false*. Justify your answers with a short proof or a counter-example. *5×2=10*

   (a) A simple graph with ten vertices has at most 45 edges only.

   (b) Any connected 2-regular graph is a cycle.

   (c) There are graphs G with $\kappa(G) < \kappa'(a)$.

   (d) Deleting some edge-cut of size 3 in the Petersen graph isolates a vertex.

   (e) Every planar graph is three-colourable.

2. (a) Let $V = \{u, v, w, x, y, z\}$ and $E = \{uv, uz, vw, wx, xy, uy, vx, wz, yz\}$. Check whether the graph G(V, E) is regular or not. If it is regular, what is the degree of regularity ? 3

(b) Find the chromatic number of the graph described in part (a). 3

(c) Define the girth of a graph and find the girth of the Petersen graph. 4

3. (a) Prove that an edge in a graph is a cut-edge if and only if it belongs to no cycle. 4

(b) A connected graph is Eulerian if and only if every vertex of it is of even degree. 6

4. (a) If G is an acyclic graph with n vertices and $n - 1$ edges, then it is a tree. 3

(b) Prove that every simple graph with at least two vertices has two vertices of equal degree. 4

(c) If G is a simple graph of diameter at least three, then prove that diameter$(\overline{G}) \leq 3$. 3

5. (a) Prove that a matching M in a graph G is a maximum matching in G if and only if G has no M-augmenting path. 5

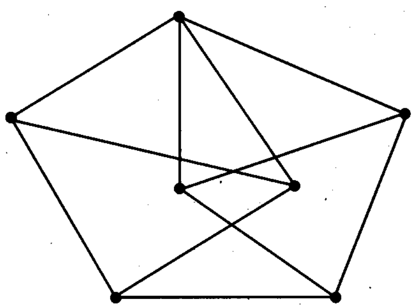
(b) If G is a simple graph, prove that $\kappa(G) \leq \kappa'(G) \leq \delta(G)$, with usual notations. 5

6. (a) For the following graph, find 4

(i) the clique number,

(ii) the independence number, and

(iii) a perfect matching, if any.



(b) Examine the planarity of the graph given in part (a) and draw a plane embedding, if it is planar. 3

(c) Define Hamiltonian closure of a graph and prove that it is well-defined. 3
