M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

00052

Term-End Examination

December, 2018

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Answer any **five** questions. Use of calculator is **not** allowed.*

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1. (a) Find the reproduction rate of the tumour cells proliferation within the tissue of animal brain with the density of cancers and local cells as $0{\cdot}2 \times 10^2$ and 4×10^{15} cells respectively. *2*

(b) Let G(t) be the amount of the glucose in the blood stream of a patient at time t. The glucose is infused into the blood stream at a constant rate of k gm/min. At the same time the glucose is converted and removed from the blood stream at a rate proportional to the amount of glucose present. If the initial concentration of the glucose in the blood stream was G_0 then find the concentration at any time t. Also, find the limiting value of the concentration. *3*

2. The population dynamics of a species is governed by the discrete model

$$x_{n+1} = x_n \exp\left[r\left(1 - \frac{x_n}{k}\right)\right]$$

where r and k are positive constants. Determine the steady-states and discuss the stability of the model. Find the value of r at which first bifurcation occurs. *5*

3. A hospital is served by two laboratories. Each laboratory has two branches and two laboratories are known to have equal share of the market as the samples arriving at each laboratory's office are at the rate of 6 per hour. The average time per test is 15 minutes. Samples arrive according to a Poisson distribution and the testing time is exponential. The two laboratories were recently bought by an investor, who is interested in consolidating them into a single laboratory to provide better service to the hospital. Comment on the new investor's proposal. *5*

4. (a) A colony of bacteria increases at a rate that is proportional to the number of bacteria in the colony. If the population quadruples in two years, find the size of the colony after eight years. 2

(b) Explain each of the following with examples :

(i) Reaction-diffusion model versus Advection-reaction-diffusion model

(ii) Hurwitz criteria

(iii) Multiple linear regression model with k predictors. 3

5. The model for interaction and diffusion of both prey and predator populations is governed by equations

$$\frac{\partial N_1}{\partial t} = a_1 N_1 - b_1 N_1 N_2 + D_1 \frac{\partial^2 N_1}{\partial x^2}$$

$$\frac{\partial N_2}{\partial t} = - d_1 N_2 + c_1 N_1 N_2 + D_2 \frac{\partial^2 N_2}{\partial x^2},$$

where $0 \leq x \leq L$

with initial and boundary conditions

$N_1(x, 0) = f_1(x) > 0$

$N_2(x, 0) = f_2(x) > 0$ for $0 \leq x \leq L$

$$\frac{\partial N_1}{\partial x} = 0 \text{ at } x = 0 \text{ and } x = L, \forall\, t$$

$$\frac{\partial N_2}{\partial x} = 0 \text{ at } x = 0 \text{ and } x = L, \forall\, t$$

The variables and parameters of the above system of equations are

N_1 = Density of prey population

N_2 = Density of predator population

a_1 = growth rate

d_1 = death rate

b_1 = predation rate

c_1 = conversion rate

D_1, D_2 = Diffusion coefficients

where a_1, d_1, b_1, c_1, D_1 and D_2 are all positive constants.

The equilibrium solutions E_0 and E_1 of the system of equations are

$$E_0 : N_1^* = 0,\ N_2^* = 0.$$

$$E_1 = \overline{N}_1 = d_1/c_1,\ \overline{N}_2 = a_1/b_1.$$

Do the stability analysis of the equilibrium point E_1. Also interpret the solution obtained. 5

6. (a) Four securities have the following expected returns :

A = 15%, B = 10%, C = 25% and D = 30%

Calculate the expected returns for a portfolio consisting of all four securities under the following conditions :

(i) The portfolio weights are 25% each.

(ii) The portfolio weights are 20% for A and C and 30% for B and D, respectively. *2*

(b) Following is the data for number of years students studied a subject and the score he/she received in that subject.

Number of years	3	4	4	2	5	3	4	5	3	2
Test score	57	78	72	58	89	63	73	84	75	48

Fit the least square line to this data. What is the score of the student who has studied for two years according to this line ? *3*