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MMT-007

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

00192

Term-End Examination
December, 2018

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no. 1 is **compulsory**. Attempt any **four** questions out of questions no. 2 to 7. Use of calculator is not allowed.*

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1. State whether the following statements are *True* or *False*. Justify your answers with the help of a short proof or a counter example. No marks will be awarded without justification. *5×2=10*

(a) The Lipschitz constant for the function

$$f(x, y) = x^2 \mid y \mid,$$

defined on $\mid x \mid \leq 1, \mid y \mid \leq 1$ is equal to 1.

(b) The Legendre Polynomial of degree n, at $x = 0$ has the value

$$P_n(0) = \begin{cases} (-1)^{\frac{n}{2}} \cdot \dfrac{1, 3, 5 \ldots (2n-1)}{2, 4, 6 \ldots 2n}, & \text{if n is even} \\ 0, & \text{if n is odd} \end{cases}$$

(c) $\mathcal{L}^{-1}\{\cot^{-1} s\} = t \sin t$

(d) The second order Runge-Kutta method when applied to IVP $y' = -100\, y$, $y(0) = 1$ will produce stable results for $0 < h < \frac{1}{50}$.

(e) The explicit scheme

$$u_i^{n+1} = u_i^n + \lambda\,[u_{i+1}^n - 2u_i^n + u_{i-1}^n],\ \lambda = \frac{k}{h^2},$$

for solving the parabolic equation $u_t = u_{xx}$ is stable for $\lambda < 1$.

2. (a) Construct the Green's function in terms of hyperbolic functions for the boundary value problem

$$y'' - k^2 y = 0,\ k \neq 0,\ y(0) = 0 = y(1).$$ 5

(b) Using the relation

$$H_n(x) = \sum_{k=0}^{[n/2]} \frac{(-1)^k \lfloor n \,(2x)^{n-2k}}{\lfloor k \ \lfloor n-2k}$$

where $\left[\frac{n}{2}\right] = \begin{cases} \frac{n}{2}, & \text{if n is odd} \\ \frac{n-1}{2}, & \text{if n is even,} \end{cases}$

find the values of $H_{2n}(0)$, $H_{2n+1}(0)$, $H'_{2n}(0)$ and $H'_{2n+1}(0)$. 5

3. (a) Find the series solutions, about x = 0, of the differential equation

$$x^2 \frac{d^2y}{dx^2} + (x + x^2)\frac{dy}{dx} + (x - a)\, y = 0 .$$ 5

(b) Reduce the second order initial value problem

$$y'' = y' + 3,\ y(0) = 1,\ y'(0) = \sqrt{3}$$

to a system of first order initial value problems. Hence find y(0·1) and y′(0·1), using Taylor series method of second order using h = 0·1. 5

4. (a) Find the solution of $\nabla^2 u = \Delta$ in R subject to the boundary conditions u(x, y) = x + y on Γ, where R is the square $0 \le x \le 1,\ 0 \le y \le 1$, using the five point formula. Assume that step length is $h = \frac{1}{3}$ along the axis. 6

(b) Find the Fourier sine transform of the function

$$f(x) = ax + x^2,\ 0 \le x \le a.$$ 4

5. (a) Show that the Bessel's function $J_n(x)$ satisfies the relation

$$J_4(x) + J_2(x) = \frac{6}{x} J_3(x)$$ 2

(b) Using Euler's method, solve

$$\frac{dy}{dx} = x - 2y, \; y(0) = 1.$$

Find y(0·5) using h = 0·1. *3*

(c) Using Laplace transform, solve the IVP

$$ty'' + y' + ty = 0, \; y(0) = 2, \; y'(0) = 0.$$

You may use the result

$$\mathcal{L}^{-1}\left[\frac{1}{\sqrt{s^2 + 1}}\right] = J_0(t) \text{ where } J_0(t)$$

is the Bessel's function of order zero. *5*

6. (a) Solve the initial value problem

$$y' = -2xy^2, \; y(0) = 1$$

with h = 0·2 on the interval [0, 0·4], using predictor-corrector method

$$P : y_{i+1} = y_i + \frac{h}{2}(3y_i' - y_{i-1}')$$

$$C : y_{i+1} = y_i + \frac{h}{2}(y_{i+1}' + y_i')$$

Perform two corrector iterations per step and use $y(x) = \frac{1}{1 + x^2}$ to obtain the starting value. *5*

(b) Using the Fourier transform, solve the initial boundary value problem

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \; -\infty < x < \infty, \; t > 0$$

$$u(x, 0) = f(x), \; \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$$ 5

7. (a) Using Crank–Nicolson's method with $h = \frac{1}{4}$, solve :

$$u_t = \frac{1}{16} u_{xx}, \; 0 < x < 1, \; t > 0$$

given $u(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = 50\,t$. 6

(b) Show the function f(x), whose Fourier cosine transform is $\frac{\sin a\alpha}{\alpha}$ is

$$f(x) = \begin{cases} 1, & 0 < x < a \\ 0, & x > a \end{cases}.$$ 4