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MMT-003

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00832

December, 2018

MMT-003 : ALGEBRA

Time : 2 hours *Maximum Marks : 50*

Note : *Question no. 1 is **compulsory**. Answer any **4** of the remaining **five** questions. **Q** denotes the field of rationals, **R** the field of real numbers and $\mathbf{Z}_p$ the finite field with p elements.*

1. Which of the following statements are *true* ? Give reasons for your answers. Marks will be given for the correct reasons only. *5×2=10*

   (a) If G is the free group generated by {a, b} and H is the subgroup generated by {a}, then H is a normal subgroup of G.

   (b) $X^2 + \bar{1}$ factors into linear factors in $\mathbf{Z}_{13}[X]$.

(c) The dimensions of all the irreducible complex representations of a group of order 49 must all be 1.

(d) If k is a field, then so is $k \times k$.

(e) The degree of $\mathbf{Q}(\omega)/\mathbf{Q}$ is 3, where ω is a primitive cube root of unity.

2. (a) Why is the polynomial $X^8 - 2$ irreducible over $\mathbf{Q}$? What is its splitting field K and what is the degree of the splitting field over $\mathbf{Q}$? Write down an element of order 2 in the Galois group of K over $\mathbf{Q}$, giving the action of the group element on a set of generators of K over $\mathbf{Q}$. 6

(b) Find all the non-isomorphic abelian groups of order 32. 4

3. (a) What is the degree of $\mathbf{Q}\left(\sqrt[3]{7}, \sqrt[5]{3}\right)$ over $\mathbf{Q}$? Justify your answer. Is the polynomial $X^5 - 5 \in \mathbf{Q}[X]$ irreducible over $\mathbf{Q}\left(\sqrt[3]{7}\right)$? Give reasons for your answer. 6

(b) Let $G = A_4$, and H be the cyclic subgroup generated by the permutation (123). Let G/H be the set of left cosets of H in G. What is the natural action of G on G/H ? Determine all the elements of the stabiliser of (12)(34) H under this action. Further, what is the cardinality of the orbit of (12)(34) H ? *4*

4. (a) Let $S = \dfrac{\mathbf{Z}_5[X]}{(X^3 + X + \bar{1})}$. How many elements does S have ? Justify your answer. Is S a field ? Justify your answer. *3*

(b) Determine the conjugacy classes of A_5 and the class equation for A_5. *5*

(c) Check whether or not (W, +) is a free semigroup, where W is the set of whole numbers. *2*

5. (a) Use the Sylow theorems to show that a group of order pq where p and q are prime numbers $p < q$, $p \nmid (q - 1)$ must be cyclic. Give an example to show that if p divides $(q - 1)$, then the group of order pq may not be cyclic. *8*

(b) If a stands for a digit between 0 and 9, give one value of a for which 8278a19051 is a valid ISBN number. *2*

6. (a) Let X = $\mathbf{Z}_2^n$. Define a subset of X to be a block if it has 4 elements that add up to **O** in $\mathbf{Z}_2^n$. Find the values of the parameters v, k, λ for this design, where $\tau = 3$. Further, if $\tau = 2$, what will the values of these parameters be ? *4*

(b) Complete the following character table of a group of order 12 :

	1 x_1	3 x_2	4 x_3	4 x_4
χ_1	1	1	ω^2	ω
χ_2	3	-1	0	0

where ω is a primitive cube root of unity. *6*