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MMT-002

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

00952

December, 2018

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Question no.* ***5*** *is* ***compulsory****. Answer any* ***three*** *questions from questions no.* ***1*** *to* ***4****. Use of calculators is* ***not*** *allowed.*

1. Consider the matrix

$$B = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}$$

(i) Check whether the matrix is positive definite or not.

(ii) Write the Jordan canonical form of B.

(iii) Find a positive semi-definite matrix A such that $A^2 = B$. *1+1+3*

2. (a) Determine a quadratic polynomial that best fits the points :

(0, 0), (1, 1), (2, 5), (3, 8). *4*

(b) Prove that if A and B are similar matrices, then their traces are the same. *1*

3. (a) Construct a QR-decomposition for

$$X = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}.$$ *3*

(b) Let T be the linear transformation from $\mathbf{R}^3$ to $\mathbf{R}^3$ given by the matrix $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Find the generalised eigenspaces $E_T(2)$ and $E_T^2(2)$. *2*

4. (a) Let $\mathbf{P}_2(\mathbf{R})$ be the real vector space of polynomials of degree ≤ 2. Let

$T : \mathbf{P}_2(\mathbf{R}) \to \mathbf{P}_2(\mathbf{R}) : T(a + bx + cx^2) = (a - c) + (a + c)x + bx^2$. Find $[T]_B$, where $B = \{x^2 + x + 1, x^2 + x, x^2\}$ is a basis of $\mathbf{P}_2(\mathbf{R})$.

Further, if B' is another basis of $\mathbf{P}_2(\mathbf{R})$, how are $[T]_B$ and $[T]_{B'}$ related ? *3*

(b) Consider the predator-prey system given by

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} 0{\cdot}35 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \end{bmatrix}$$

where x_k and y_k are the populations of the predators and the prey, respectively, at time k. What is the long-term behaviour of the population vector $\begin{bmatrix} x_k \\ y_k \end{bmatrix}$? *2*

5. Which of the following statements are *True*, and which are *False* ? Justify your answers. *5×2=10*

(a) Two $n \times n$ matrices with the same minimal polynomials have the same Jordan canonical form.

(b) If the rank of an $n \times n$ matrix is $n - 1$, then at least one of the eigenvalues of A is zero.

(c) A diagonalisable matrix is also unitarily diagonalisable.

(d) If the determinant of a matrix is positive, then the matrix is positive definite.

(e) $\forall\ A \in \{X \in \mathbf{M}_n(\mathbf{R}) \,|\, X$ is positive definite and $x_{ii} = 1\ \forall\ i = 1, \ldots, n\}$, det(A) is bounded above.
