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**BICE-027**

# B.Tech. – VIEP – MECHANICAL ENGINEERING / B.Tech. CIVIL ENGINEERING (BTMEVI / BTCLEVI)

## Term-End Examination

## December, 2018

00333

## BICE-027 : MATHEMATICS-III

*Time : 3 hours* *Maximum Marks : 70*

***Note :*** *Attempt any **two** parts from each question. Use of scientific calculator is permitted. All questions are carry equal marks.*

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1. (a) Obtain Fourier series for the function

$$f(x) = \begin{cases} x & -\pi < x < 0 \\ -x & 0 < x < \pi \end{cases}$$

and hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

(b) Develop $f(x) = \sin\left(\frac{\pi x}{\rho}\right)$ in half-range cosine series in the range $0 < x < \rho$.

(c) Obtain the Fourier series expansion of

$f(x) = \left(\frac{\pi - x}{2}\right)$, for $0 < x < 2$ *2×7=14*

2. (a) Solve Partial differential equation :

$x^2p + y^2q = (x + y) z$

(b) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x$

(c) Solve the Partial differential equation

$(D^2 - D'^2 - 3D + 3D') z = xy + e^{x + 2y}$. *2×7=14*

3. (a) A string is stretched and fastened to two points '*l*' apart. Motion is started by displacing the string in the form $y = A \sin \frac{\pi x}{\rho}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x, t) = A \sin \frac{\pi x}{\rho} \cdot \cos \frac{\pi t}{\rho}$.

(b) Find the temperature in a bar of length 2 whose ends are kept at zero and lateral surface insulated if the initial temperature is $\sin \frac{\pi x}{2} + 3 \sin \frac{5\pi x}{2}$.

(c) Solve the following equation by the method of separation of variables

$$\frac{\partial^2 u}{\partial x\, \partial t} = e^{-t} \cos x$$

given that u = 0 when t = 0 and $\frac{\partial u}{\partial t} = 0$ when x = 0. *2×7=14*

4. (a) Use separation of variables method to solve the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

subject to the boundary condition

$u(0, y) = u(l, y) = u(x, 0) = 0$ and

$u(x, a) = \sin \frac{n\pi x}{\rho}$.

(b) The diameter of a semi-circular plate of radius 'a' is kept at 0°C and the temperature at the semi-circular boundary is T°C. Show that the steady state temperature in the plate is given by

$$u(r, \theta) = \frac{4T}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \left(\frac{r}{a}\right)^{2n-1} \sin (2n-1)\, \theta.$$

(c) Use the method of separation of variables to solve the equation

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \text{ given that } u(x, 0) = 6e^{-3x}.$$

*2×7=14*

**5.** (a) Find Fourier sine transform of $\frac{e^{-ax}}{x}$, $a > 0$.

Hence, find Fourier sine transform of $\frac{1}{x}$.

(b) Find the Fourier transform of

$$F(x) = \begin{cases} 1, & |x| < a \\ 0 & |x| > a \end{cases} \quad \text{and hence}$$

evaluate $\int_{-\infty}^{\infty} \frac{\sin ap \cos px}{p} \, dp$

(c) Find the Fourier cosine transform of $e^{-x^2}$.

*2×7=14*