No. of Printed Pages : 6 **BME-001**

B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination

00293

December, 2018

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**. Use of statistical tables and calculator is permitted.*

1. Answer any ***five*** of the following : *5×4=20*

   (a) Evaluate

   $$\lim_{x \to 0} x^x.$$

   (b) If $y = \tan^{-1}\left(\sqrt{1+x^2} - x\right)$, compute $\frac{dy}{dx}$.

   (c) If $v = f\left(\frac{x}{z}, \frac{y}{z}\right)$,

   prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + z\frac{\partial v}{\partial z} = 0$.

(d) If $u = x + y + z$, $y + z = uv$, $z = uvw$, find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.

(e) Solve the differential equation

$(1 - \sin x \tan y)\, dx + (\cos x \sec x^2)\, dy = 0$.

(f) Solve the differential equation

$\cos x \frac{dy}{dx} + y \sin x = 1$.

2. Answer any ***four*** of the following : *4×4=16*

(a) Find a unit normal vector of the surface $xy^2 + 2yz = 8$ at the point $(3, -2, 1)$.

(b) Show that the vector

$\vec{v} = (2x + 3y)\hat{i} + (x - y)\hat{j} - (x + y + z)\hat{k}$

is solenoidal.

(c) Find the directional derivative of $f(x, y, z) = xy^2 + 4xyz + z^2$ at the point $(1, 2, 3)$ in the direction of $3\hat{i} + 4\hat{j} - 5\hat{k}$.

(d) Evaluate the integral $\iint_S y\, dA$ where S is the portion of the cylinder $x = 6 - y^2$ in the first octant bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $z = 8$.

(e) Use the divergence theorem to evaluate $\iint_S (\vec{v} \cdot \hat{n})\, dA$, where

$\vec{v} = x^2 z\hat{i} + y\hat{j} - xz^2\hat{k}$ and S is the boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y$.

(f) Evaluate the integral $\iint_S (\nabla \times \vec{v}).\hat{n}\, dA$ by Stokes theorem where

$\vec{v} = 2yz\hat{i} + 3zx\hat{j} + xy\hat{k}$, S is the paraboloid $z = x^2 + y^2$ for $x^2 + y^2 \leq 4$.

3. Answer any ***six*** of the following : *6×3=18*

(a) Find the adjoint and inverse of

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}.$$

(b) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}.$$

(c) Find P and Q such that the normal form of

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$

(d) Test if the system is consistent or inconsistent. If consistent, then find the solution.

$$-x_1 + x_2 + 2x_3 = 2$$

$$3x_1 - x_2 + x_3 = 6$$

$$-x_1 + 3x_2 + 4x_3 = 4$$

(e) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$

(f) Verify the Cayley-Hamilton theorem and find the inverse of matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}.$$

(g) Show that

$$A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

is skew-Hermitian.

(h) Solve the following equations by using Cramer's rule :

$x + y + z = 11$

$2x - 6y - z = 0$

$3x + 4y + 2z = 0$

4. Answer any ***four*** of the following : *4×4=16*

(a) Find the probability that at least two 9's appear (as a sum) in four tosses of a pair of fair dice.

(b) A class has 10 boys and 5 girls. Three students are selected at random, one after the other. Find the probability that (i) the first two are boys and the third is a girl, (ii) the first and the third are boys and the second is a girl.

(c) A fair die is tossed 7 times. Determine the probability that a 5 or a 6 appears (i) exactly 3 times (ii) never occurs.

(d) Suppose 300 misprints are distributed randomly throughout a book of 500 pages. Find the probability P that a given page contains (i) exactly two misprints (ii) two or more misprints.

(e) Is there reason to believe that the life expectancy in South and North India is same or not from the following data :

South	North
34	49·7
39·2	55·4
46·1	57·0
48·7	54·2
49·4	50·4
45·9	44·2
55·3	53·4
42·7	57·5
43·7	61·9
	56·6
	58·2

(f) A company claims that the mean thermal efficiency of diesel engines produced by them is 32·3%. To test this claim, a random sample of 40 engines was examined which showed the mean thermal efficiency of 31·4% and standard deviation of 1·6%. Can the claim be accepted or not, at 0·01 level of significance ?