No. of Printed Pages : 5

ET-102

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering)

Term-End Examination

December, 2018

00043

ET-102 : MATHEMATICS – III

Time : 3 hours *Maximum Marks : 70*

Note : *Question no. 1 is* ***compulsory****. Attempt any other* ***eight*** *questions from question nos. 2 to 15. Use of calculator is allowed.*

---

1. Fill in the blanks. ***All*** parts are to be attempted. *7×2=14*

(a) The series $1-\frac{1}{2^p}+\frac{1}{3^p}-\frac{1}{4^p}+\frac{1}{5^p}\ldots$ is conditionally convergent when the value of p is ________.

(b) Let $\sum_n x_n$ be a positive term series such that $\underset{n\to\infty}{\mathrm{Lt}}(x_n/x_{n+1})=e$, then the test fails to provide a definite information about convergence or divergence for ________.

(c) A function f(x) is defined in the interval $0 \leq x \leq \pi$ and if we take $f(-x) = f(x)$ in $-\pi \leq x < 0$, then we get an ________ function for which Fourier series coefficients ________ are zero.

(d) If f(x, y) and $\frac{\partial}{\partial y} f(x, y)$ are continuous in a closed bounded region D in xy-plane, then IVP $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$, has a unique solution in the interval ________.

(e) The Laplace Transform of $\frac{\sin t}{t}$ is ________ given that $\mathcal{L}\left\{\frac{\sin 2t}{t}\right\} = \tan^{-1}\frac{1}{s}$.

(f) The solution of differential equation $(D^2 - 2D + 2)^2 y = 0$ is ________.

(g) The residue of $f(z) = \frac{z+2}{(z+1)^2 (z-2)}$ at pole $z = 2$ is ________.

2. Use Laplace Transform to solve the differential equation

$$y'' + 9y = t \text{ with } y(0) = 0,\ y(\pi/2) = 0.$$ 7

3\. (a) Test the series $\sum (-1)^n \sin\left(\frac{1}{n}\right)$ for absolute convergence. $3\frac{1}{2}$

(b) Test, for convergence, the series

$$\sum \frac{1.3.5.\dots(2n-1)}{2.4.6.\dots(2n)}$$ $3\frac{1}{2}$

4\. Find a series of cosines of multiples of x which will represent x sin x in the interval $(0, \pi)$. 7

5\. An alternating current, after passing through a rectifier has the form

$$i(x) = \begin{cases} I_0 \sin x & \text{for} \quad 0 \le x \le \pi \\ 0 & \text{for} \quad \pi \le x \le 2\pi \end{cases},$$

where I_0 is the maximum current and the period is 2π. Express i(x) in a Fourier Series. 7

6\. Find :

$$\mathcal{L}^{-1}\left[\frac{s-1}{(s+3)(s^2+2s+2)}\right]$$ 7

7\. Evaluate, using complex function and residue theory,

$$\int_{-\infty}^{\infty} \frac{\sin x}{(x^2+x+1)^2}\, dx.$$ 7

8. (a) Prove that

$$\int_0^{\pi} \frac{a\, d\theta}{a^2 + \sin^2 \theta} = \frac{\pi}{\sqrt{1 + a^2}}, a > 0.$$ *4*

(b) Find the bilinear transformation that maps $i, 1, -1$ into $1, 0, \infty$. *3*

9. Find a series solution, near $x = 0$, of the differential equation

$$9x(1 - x)y'' - 12y' + 4y = 0.$$ *7*

10. If $y = x$ and $y = xe^{2x}$ are the two solutions of complementary function of the differential equation

$$x^2 y'' - 2x(1 + x)y' + 2(1 + x)y = x^3,$$

use method of variation of parameters to find its particular integral. *7*

11. (a) Determine the analytic function $f(z) = u + iv$ if $u - v = (x - y)(x^2 + 4xy + y^2)$. *4*

(b) Find Laurent series for

$$f(z) = \frac{7z - 2}{(z + 1)(z)(z - 2)}$$

in the annulus $0 < |z + 1| < 1$. *3*

12. Find the temperature $u(x, t)$ in a box of length π, which is perfectly insulated at the ends $x = 0$ and $x = \pi$, assuming $c = 1$ and $u(x, 0) = x^2$. 7

13. Solve the partial differential equation

$$(D_x^2 - D_x D_y + D_y - 1) z = \cos(x + 2y) + e^y + xy + 1. \quad 7$$

14. Find the characteristic function, transfer function, frequency response function and characteristic roots of the equation

$$(D^2 + 2D + 7) x = f:$$

Test it for stability. 7

15. Find the natural period of a mass-dashpot-spring system if the mass weight is 15 lbs and stretches a steel spring 3 inches. If the spring is stretched an additional 3 inches and then released, determine the subsequent motion. 7