No. of Printed Pages : 6

ET-101(A)

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) / BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI

Term-End Examination

00293

December, 2018

ET-101(A) : MATHEMATICS – I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**. Use of scientific calculator is allowed.*

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1. Answer any ***five*** of the following : *5×4=20*

(a) If $f(x) = \sqrt{9 - x^2}$ then compute

$$\lim_{x \to 2} \frac{f(2) - f(x)}{x - 2}$$

(b) Evaluate

$$\lim_{x \to 0} \frac{1 - \cos 2x}{x}$$

(c) If $y = (\tan x)^{\log x}$, then compute $\frac{dy}{dx}$.

(d) If $\sin y = x \sin (a + y)$, then prove that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

(e) Find $\frac{dy}{dx}$, when

$x = a (t + \sin t)$ and $y = a (1 - \cos t)$.

(f) Solve :

$$(1 + x^2)\, dy = (1 + y^2)\, dx$$

(g) Determine the value of K for which

$$f(x) = \begin{cases} \dfrac{\sin 5x}{3x} & \text{if } x \neq 0 \\ K & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$.

(h) If $y = x + e^x$, then prove that

$$\frac{d^2x}{dy^2} = -\frac{e^x}{(1 + e^x)^3}$$

2. Answer any ***four*** of the following : *4×4=16*

(a) Evaluate

$$\int \sqrt{1 - \sin 2x}\, dx$$

(b) Evaluate

$$\int \frac{3x^2}{1+x^6}\,dx$$

(c) Compute

$$\int_2^4 \frac{1}{x}\,dx$$

(d) Find the area of the region bounded by the curve $y = x - x^2$ between $x = 0$ and $x = 1$.

(e) Evaluate

$$\int_0^1 \frac{1}{1+x^2}\,dx$$

by using Simpson's $\frac{1}{3}$ rule taking $h = \frac{1}{4}$.

(f) Compute the area bounded by the curve $y^2 = 9x$ and the lines $x = 1$, $x = 4$ and $y = 0$.

3. Answer any ***four*** of the following : *4×4=16*

(a) Find a unit vector parallel to the sum of the vectors

$$\vec{r}_1 = 2\hat{i} + 4\hat{j} - 5\hat{k} \text{ and}$$

$$\vec{r}_2 = \hat{i} + 2\hat{j} + 3\hat{k}.$$

(b) If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$, find 't' such that $\vec{a} + t\vec{b}$ is perpendicular to $\vec{c}$.

(c) Constant forces $\vec{P} = 2\hat{i} - 5\hat{j} + 6\hat{k}$ and $\vec{Q} = -\hat{i} + 2\hat{j} - \hat{k}$ act on a particle. Determine the work done when the particle is displaced from A to B, the position vectors of A and B being $4\hat{i} - 3\hat{j} - 2\hat{k}$ and $6\hat{i} + \hat{j} - 3\hat{k}$ respectively.

(d) A particle moves so that its position vector is given by

$$\vec{r} = \cos \omega t\,\hat{i} + \sin \omega t\,\hat{j}.$$

Show that the velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$ and $\vec{r} \times \vec{v}$ is a constant vector.

(e) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that

(i) $\text{div}\ \vec{r} = 3$,

(ii) $\text{curl}\ \vec{r} = \vec{0}$.

(f) Show that the vector field

$$\vec{v} = (\sin y + z)\hat{i} + (x \cos y - z)\hat{j} + (x - y)\hat{k}$$

is irrotational.

4. Answer any *six* of the following : *6×3=18*

(a) Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

(b) If

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 4 & 1 & -1 \\ 1 & 2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 1 & 3 \\ 2 & 1 & 1 \\ -1 & 5 & -3 \end{bmatrix}$$

find 3A – 4B.

(c) If

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

show that $[2I - A][10I - A] = 9I$.

(d) If

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}, \text{ find } A^{-1}.$$

(e) Express the following matrix as the sum of a symmetric and a skew-symmetric matrix :

$$\begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$$

(f) Solve the following system of equations by matrix method :

$$x + y + z = 6$$

$$x - y + 2z = 5$$

$$3x + y + z = 8$$

(g) Find x, y, z and w

if $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$

(h) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.