No. of Printed Pages : 3

MMTE-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00981

Term-End Examination

December, 2017

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Answer any **four** questions from questions no. 1 to 5. Question no. **6** is **compulsory**.*

1. (a) Let C be the code generated by the matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ over } \mathbf{F}_3.$$

(i) How many codewords will C have, and why ?

(ii) Give three distinct codewords of C and find their Hamming weights.

(iii) List all the steps required for finding the minimum distance of any code. *7*

(b) Write the generator matrix for the Reed-Muller code R(2, 4). *3*

2. (a) Give the minimal polynomial, over $\mathbf{F}_2$, of each element of $\mathbf{F}_8$. *6*

(b) Let C be a non-zero cyclic code in $\mathbf{R}_n$. Prove that there exists $g(x) \in C$ such that $g(x)$ divides $(x^n - 1)$. *4*

3. (a) Construct a BCH code over $\mathbf{F}_3$ of length 13 and design distance 2 with the primitive element α satisfying $x^3 + 2x + 1 = 0$. Justify each step of your construction. *5*

(b) Let C be an extended binary Golay code. Show that $A_0 = A_{24} = 1$, and find A_n where n is not divisible by 4 and $n < 24$. *2*

(c) Check whether or not there are self-dual, extended cyclic binary codes of lengths 7 and 17. *3*

4. (a) Find the weight distribution and weight enumerator of the code C generated by the matrix $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ over $\mathbf{F}_2$. *3*

(b) Let C be the code generated by the matrix

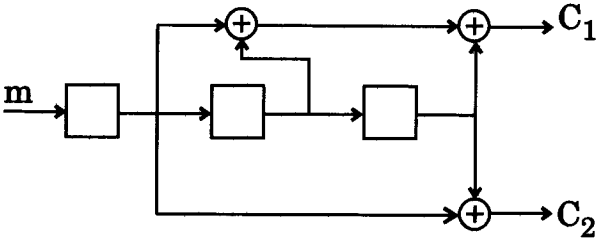
$$G = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$ over $\mathbf{Z}_4$.

(i) List all codewords of C.

(ii) What is the minimum distance of C ?

(iii) List all codewords of the Gray image of C. *7*

5. (a) Find the convolutional code for the message 11011. The convolutional encoder is given below. *5*



(b) Describe the Viterbi Decoding Algorithm, with an example. *5*

6. Which of the following statements are *True* and which are *False* ? Give reasons for your answers. *10*

(a) Every binary Hamming code is a cyclic code.

(b) $R_{q,n} = \mathbf{F}_q[X] / \langle x^n - 1 \rangle$ is a field if and only if $n = 1$.

(c) The degree of a generator polynomial of a cyclic code is equal to the dimension of the code.

(d) There is no self-dual code of length 5.

(e) There is no (5, 3, 4) LDPC code.