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**MMT-007**

# M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
# M.Sc. (MACS)

01321

## Term-End Examination

## December, 2017

## MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

*Time : 2 hours* *Maximum Marks : 50*
*(Weightage : 50%)*

**Note :** *Question no. 1 is* ***compulsory****. Attempt any* ***four*** *questions out of questions no. 2 to 7. Use of non-programmable scientific calculator is allowed.*

**1.** State whether the following statements are *True* or *False*. Justify your answers with the help of a short proof or a counter-example. *5×2=10*

(a) For Legendre polynomial $P_n(x)$ of degree n,

$$P_n(-1) = 1.$$

(b) If f(t) is continuous and possesses continuous derivatives and both f(t) and f′(t) are of exponential order as $t \to \infty$, then

$$\mathcal{L}[f'(t)] = \mathcal{L}[f(t)] - sf(0),$$

where $\mathcal{L}$ denotes Laplace Transform of the function.

(c) For the test equation $y' = \lambda y$, $y(x_0) = y_0$, if $\lambda < 0$, we consider absolute stability and if $\lambda > 0$, we consider relative stability.

(d) If the given partial differential equation is self-adjoint, then application of variation principle and the Galerkin method lead to a different matrix system.

(e) For the differential equation

$$x^3y'' + xy' + 2y = 0,$$

$x = 0$ is irregular singular point.

**2.** (a) Find the Fourier cosine transform of the function

$$f(x) = \begin{cases} 3x & 0 < x < \frac{1}{3} \\ 2 - x & \frac{1}{3} < x < 1 \\ 0 & x > 1 \end{cases}.$$ *3*

(b) Using generating function of Hermite polynomial $H_n(x)$, expand $f(x) = 3x^3 - 4x^2 + 6x$ in a series of the form

$$\sum_{n=0}^{\infty} a_n H_n(x).$$ *5*

(c) If $f'(x_k)$ is approximated by

$$h\ f'(x_k) = a\ f(x_{k+1}) + b\ f(x_{k-1}),$$

find the values of a and b. What is the order of approximation ? *2*

3. (a) Use second order finite difference method to solve the boundary value problem

$$y'' - 3y' + 2y = 0$$

with $2y(0) - y'(0) = 1$,

$y(1) + y'(1) = 2e + 3e^2$ and $h = \frac{1}{2}$.

Set up the system of equations to obtain the solution. 5

(b) Solve the initial value problem $y' = -2xy^2$, $y(0) = 1$ with $h = 0{\cdot}2$ on the interval $[0, 0{\cdot}4]$ using predictor-corrector method, with predictor P and corrector C as given below :

$$P : y_{k+1} = y_k + \frac{h}{2}(3y'_k - y'_{k-1})$$

$$C : y_{k+1} = y_k + \frac{h}{2}(y'_{k+1} + y'_k)$$

Perform two corrector iterations per step. Use the exact solution of $y(x) = \frac{1}{1+x^2}$ to obtain the starting value. 3

(c) For Bessel's function $J_n(n)$, show that

$$\frac{d}{dx}(x^{-n} J_n) = -x^{-n} J_{n+1}.$$ 2

4. (a) Using the Frobenius method, find the power series solution near $x = 0$ of the differential equation

$$x^2 \frac{d^2y}{dx^2} + (x + x^2)\frac{dy}{dx} + (x - 9)\,y = 0.$$ 6

(b) If $f(x) = 0, \quad -1 < x \leq 0$

$= x, \quad 0 < x < 1,$

show that

$$f(x) = \frac{1}{4} P_0(x) + \frac{1}{2} P_1(x) + \frac{5}{16} P_2(x) - \frac{3}{32} P_4(x) + \dots$$

where $P_n(x)$ is a Legendre polynomial of degree n. 4

5. (a) Using the five-point formula and assuming the uniform step length $h = \frac{1}{3}$ along the axes, find the solution of $\nabla^2 u = x^2 + y^2$ in R, where R is the triangle $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq x + y \leq 1$. On the boundary of the triangle $u(x, y) = x^2 - y^2$. 6

(b) Solve the integral equation

$$\int_0^\infty f(x) \sin \alpha x \, dx = \begin{cases} 2 - 3\alpha, & 0 < \alpha < 1 \\ 0, & \alpha > 1 \end{cases}.$$ 2

(c) Explain the difference between explicit single-step method and implicit single-step method for solving the IVP

$$\frac{dy}{dx} = f(x, y), \; y(x_0) = y_0.$$ 2

6. (a) Solve the partial differential equation,

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, x > 0, t > 0$$

given that u(0, t) = 10 sin 2t,

$$u(x, 0) = 0 = u_t(x, 0),$$

$$\lim_{x \to \infty} u(x, t) = 0,$$

using Laplace Transforms. 5

(b) Using the Schmidt method, with $\lambda = \frac{1}{6}$, find the solution of the initial value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},$$

subject to the conditions

$$u(0, t) = 0 = u(1, t)$$

$$\text{and } u(x, 0) = \begin{cases} 2x & \text{for } x \in \left[0, \frac{1}{2}\right] \\ 2(1 - x) & \text{for } x \in \left[\frac{1}{2}, 1\right] \end{cases}.$$

Take h = 0·2 and integrate for two time levels. 5

7. (a) Find the solution of the problem $\nabla^2 u = 0, 0 \leq x \leq 1, 0 \leq y \leq 1, u = x + y$, on the boundary using Galerkin method with triangular elements and one internal node ($h = \frac{1}{2}$). 6

(b) Using convolution theorem, evaluate

$$\mathcal{L}^{-1} = \left\{\frac{1}{(s + 1)(s^2 + 1)}\right\},$$

where $\mathcal{L}^{-1}$ is inverse Laplace Transform. 4