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MMT-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

00471 **December, 2017**

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

Note : *Question no. 1 is **compulsory**. Attempt any **three** questions from questions no. 2 to 5. Use of calculator is **not** allowed.*

1. State, giving reasons, whether the following statements are *True* or *False* : *5×2=10*

(a) $f(z) = \sinh z \cosh z$ is a bounded function.

(b) The curve defined by $z(t) = \cos t, -\pi \leq t \leq \pi$ is a Jordan curve.

(c) If $f(z) = \text{Log } z$, then $z = 0$ is an isolated singular point of $f(z)$.

(d) If T be a linear fractional transformation such that $T(0) = 0$ and $T(\infty) = \infty$, then $T(z) = \alpha.z$ for some non-zero complex number α.

(e) If $f(z) = \tan z$, then $\oint_C f(z)\,dz = 0$, where $C : |z| = 1$.

2. (a) Find the zeros and singularities of the function $f(z) = \frac{z}{4\cos^2 z - 1}$ in $|z| \leq 1$. Also find the residue at the poles. *2*

(b) Expand $f(z) = \frac{1}{(z-1)^2 (z-3)}$ in a Laurent series valid for $0 < |z-3| < 2$. *3*

3. (a) Consider the region $R = \{z : |z| \leq 2\}$. If $f(z) = 2 - z$ in R, then find a point in R where $|f(z)|$ attains its maximum value. *2*

(b) Find the harmonic conjugate $v(x, y)$ of the harmonic function $u(x, y) = xy + x + 2y - 5$ and also an analytic function $f(z) = u + iv$, if $f(4i) = 3 + 13i$. *3*

4. (a) Find the image of the lines $y = C_2$, $(C_2 > 0)$ under the mapping $w = z^2$. Identify the curve represented by the image. *2*

(b) Find all solutions to the equation $\sin z = 5$. *3*

5. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3}$ using contour integration. *5*