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MMT-003

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00001

Term-End Examination

December, 2017

MMT-003 : ALGEBRA

Time : 2 hours *Maximum Marks : 50*

(Weightage : 70%)

Note : *Question no.* ***6*** *is* ***compulsory****. Attempt any* ***four*** *questions from questions no. 1 to 5.*

1. (a) The table below is a partial character table of a finite group in which $\alpha = \frac{1}{2}(-1 + i\sqrt{3})$ and $\beta = \frac{1}{2}(-1 + i\sqrt{7})$. The conjugacy classes are all shown.

	(1)	(3)	(3)	(7)	(7)
χ_1	1	1	1	α	$\bar{\alpha}$
χ_2	3	β	$\bar{\beta}$	0	0
χ_3	3	$\bar{\beta}$	β	0	0

(i) Determine the order of the group, the number of irreducible representations and their dimensions.

(ii) Determine the remaining characters. *8*

(b) Check whether 978-81-265-3228-5 is a valid ISBN number or not. *2*

2. (a) Let ω and α denote the primitive third and sixth roots of unity, respectively. Prove that $\mathbf{Q}(\alpha) = \mathbf{Q}(\omega)$. Further, obtain $[\mathbf{Q}(\alpha) : \mathbf{Q}]$. *5*

(b) Let $H = \{z \mid Im(z) > 0\}$. Check whether $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot z = \dfrac{az+b}{cz+d}$ defines an action of $SL_2(\mathbf{Z})$ on H or not. Further, if it is an action, find the stabiliser of i. If the given function is not an action, then define an action of $SL_2(\mathbf{Z})$ on H. *5*

3. (a) Find the splitting field K of $x^4 - 3$ over $\mathbf{Q}$. Also find $[K : \mathbf{Q}]$. Further, does K contain a subfield which is not normal over $\mathbf{Q}$? Give reasons for your answer. *6*

(b) Let $\pi : \mathbf{Z} \to \dfrac{\mathbf{Z}}{4} \times \dfrac{\mathbf{Z}}{7} \times \dfrac{\mathbf{Z}}{9}$ be the natural homomorphism defined by $\pi(x) = (x \pmod 4), x \pmod 7, x \pmod 9))$. Find a pre-image of $(1 \pmod 4), 2 \pmod 7), 3 \pmod 9))$ lying in $[50, 200]$. *4*

4. (a) Show that an operation $*$ can be defined on any non-empty set S such that $(S, *)$ is a semigroup. *3*

(b) De... a non-trivial two-dimensional comple... ...resentation of D_{10}. Check all the propert... ...quired. *4*

(c) Does $SU_2(\mathbf{C})$ act transitively on $\mathbf{C}^2 \setminus \left\{\begin{pmatrix}0\\0\end{pmatrix}\right\}$? Justify your answer. *3*

5. (a) Find a Sylow-p subgroup of $GL_2(\mathbf{F}_p)$. *5*

(b) Show that if there is a Steiner system S(3, 6, n), then $n \equiv 2 \pmod{20}$ or $n \equiv 6 \pmod{20}$. *5*

6. State whether the following statements are *True* or *False*. Give reasons for your answers. *10*

(a) If G is an infinite group acting on a set S, then $|O_s|$ cannot be finite for any $s \in S$.

(b) If G is a free group, then so is $\frac{G}{H}$, where $H \trianglelefteq G$.

(c) $SL_4(\mathbf{R}) \subseteq SP_4(\mathbf{R})$.

(d) If F is a field, so is $F \times F$.

(e) Every non-trivial representation of a finite group is faithful.