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MMT-002

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

02941

Term-End Examination

December, 2017

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours

Maximum Marks : 25
(Weightage : 70%)

Note : *Question no.* ***5*** *is* ***compulsory****. Answer any* ***three*** *questions from questions no. 1 to 4. Use of calculators are* ***not*** *allowed.*

1. (a) Let $\beta = \{u_1, u_2, u_3\}$ be an ordered basis of $\mathbf{R}^3$ and let the matrix of a linear operator T on $\mathbf{R}^3$ with respect to this basis be

$$[T]_\beta = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Find the matrix of T with respect to the basis $\{u_1 + u_2, u_2 + u_3, u_3\}$. *2*

(b) Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 2 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Check whether the system $Ax = \mathbf{b}$ is inconsistent or not. If it is, find a least squares solution for $Ax = \mathbf{b}$. If it is not inconsistent, obtain an SVD for A. *3*

2. (a) Check whether the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

is diagonalisable. If it is diagonalisable, find an invertible matrix P so that $P^{-1} A P$ is a diagonal matrix. Otherwise, obtain the Jordan canonical form of A. *3*

(b) Find the square root of the matrix $\begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$. *2*

3. (a) What are the singular values of the matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}?$$ *1*

(b) Solve the system of differential equations $\frac{dy(t)}{dt} = A\, y(t)$, with $y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} -8 & 25 \\ -4 & 12 \end{bmatrix}$. *4*

4\. (a) Write all possible Jordan canonical forms for a 4×4 matrix whose only distinct eigenvalues are 1 and 2, the geometric multiplicity of 1 is two and the minimal polynomial is of degree 3. *2*

(b) Obtain a QR-decomposition for the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$ *3*

5\. Which of the following statements are *true*, and which are not ? Give reasons for your answers. *10*

(a) If T is a linear operator on a finite-dimensional vector space whose matrix with respect to a basis is a nilpotent matrix, then T is not onto.

(b) Two $n \times n$ matrices with the same trace are similar.

(c) There is a unitary matrix with one of the entries equal to 2.

(d) If A is positive definite, then A^{-1} exists and is positive definite.

(e) If a matrix has a generalised inverse, then it is invertible.
