

## MCA (Revised)

## Term-End Examination

00270

December, 2017

## MCS-033 : ADVANCED DISCRETE MATHEMATICS

Time : 2 hours

Maximum Marks : 50

**Note :** Question no. 1 is **compulsory**. Attempt any **three** questions from the rest.

1. (a) Using induction, verify that

$$\sqrt{5} f_n = \left( \frac{1 + \sqrt{5}}{2} \right)^n - \left( \frac{1 - \sqrt{5}}{2} \right)^n, n \geq 1$$

where  $f_n = f_{n-1} + f_{n-2}$  and  $f_0 = 0$  and  $f_1 = 1$ . 5

- (b) Determine the number of subsets of a set of  $n$  elements, where  $n \geq 0$ . 5
- (c) Find the sum of the series

$$\sum_{k=0}^{\infty} \frac{(k+1)^2}{k} = \frac{1^2}{0} + \frac{2^2}{1} + \dots + \frac{(n+1)^2}{n} + \dots$$

using exponential generating functions. 5

- (d) Take three vertices  $x, y, z$  and draw all possible  $(3, 2)$  graphs on these vertices. 5
2. (a) Find the number of integer solutions of the linear equation
- $$a_1 + a_2 + \dots + a_k = n,$$
- using generating function techniques, when  $a_i \geq 0$ . 5
- (b) State and prove the handshaking theorem. 5
3. (a) Solve the recurrence relation  $a_{n+1}^2 = 5a_n^2$  where  $a_n > 0$  and  $a_0 = 2$ . 5
- (b) Construct a 5 regular graph on 10 vertices. 5
4. (a) Solve the linear recurrence
- $$a_n - a_{n-1} = f_{n+2} \cdot f_{n-1} \quad n \geq 1$$
- where  $a_0 = 2$  and  $f_i$  denotes the  $i^{\text{th}}$  Fibonacci number. 5
- (b) Show that for a subgraph  $H$  of a graph  $G$ ,  $\Delta(H) \leq \Delta(G)$ . 5
5. (a) Find all the graphs that have edge chromatic number 1. 5
- (b) Show that  $C_6$  is bipartite and  $K_3$  is not bipartite. 5