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BCS-012

BACHELOR OF COMPUTER APPLICATIONS (BCA) (Revised)

Term-End Examination

10403

December, 2017

BCS-012 : BASIC MATHEMATICS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number 1 is* ***compulsory****. Attempt any* ***three*** *questions from the rest.*

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1. (a) Show that

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$$ 5

(b) Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$.

Show that $f(A) = O_{2\times2}$. Use this result to find A^5. 5

(c) Find the sum up to n terms of the series

$$0{\cdot}4 + 0{\cdot}44 + 0{\cdot}444 + \ldots$$ 5

(d) If 1, ω, ω^2 are cube roots of unity, show that $(1 + \omega)(1 + \omega^2)(1 + \omega^3)(1 + \omega^4)(1 + \omega^6)(1 + \omega^8) = 4$. 5

(e) If $y = ae^{mx} + be^{-mx} + 4$, show that

$$\frac{d^2y}{dx^2} = m^2(y - 4).$$ 5

(f) A spherical balloon is being inflated at the rate of 900 cubic centimetres per second. How fast is the radius of the balloon increasing when the radius is 25 cm ? 5

(g) Find the value of λ for which the vectors $\vec{a} = 2\hat{i} - 4\hat{j} + 3\hat{k}$, $\vec{b} = \lambda\hat{i} - 2\hat{j} + \hat{k}$, $\vec{c} = 2\hat{i} + 3\hat{j} + 3\hat{k}$ are co-planar. 5

(h) Find the angle between the pair of lines

$$\frac{x-5}{2} = \frac{y-3}{3} = \frac{z-1}{-3} \text{ and}$$

$$\frac{x}{3} = \frac{y-1}{2} = \frac{z+5}{-3}.$$ 5

2. (a) Solve the following system of equations by using matrix inverse : 5

$3x + 4y + 7z = 14$, $2x - y + 3z = 4$, $x + 2y - 3z = 0$

(b) Show that $A = \begin{bmatrix} 3 & 4 & -5 \\ 2 & 2 & 0 \\ 1 & 1 & 5 \end{bmatrix}$ is row equivalent to I_3. 5

(c) Use the principle of mathematical induction to prove that

$$1^3 + 2^3 + \dots + n^3 = \frac{1}{4} n^2 (n+1)^2$$

for every natural number n. 5

(d) Find the quadratic equation with real coefficients and with the pair of roots $\frac{1}{5 - \sqrt{72}}, \frac{1}{5 + 6\sqrt{2}}$. 5

3. (a) How many terms of the G.P. $\sqrt{3}, 3, 3\sqrt{3}, \dots$ add up to $120 + 40\sqrt{3}$? 5

(b) If $\left(\frac{1-i}{1+i}\right)^{10} = a + ib$, then show that a = 1 and b = 0. 5

(c) Solve the equation $8x^3 - 14x^2 + 7x - 1 = 0$, the roots being in G.P. 5

(d) Solve the inequality $\left|\frac{x-4}{2}\right| \leq \frac{5}{12}$ and graph the solution set. 5

4. (a) Determine the values of x for which the following function is increasing and for which it is decreasing : 5

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$$

(b) Show that $f(x) = 1 + x^2 \ln\left(\frac{1}{x}\right)$ has a local maximum at $x = \frac{1}{\sqrt{e}}$, $(x > 0)$. 5

(c) Evaluate the integral

$$\int \frac{dx}{1 + 3e^x + 2e^{2x}}.$$ 5

(d) Find the length of the curve $y = \frac{2}{3} x^{3/2}$ from $(0, 0)$ to $\left(1, \frac{2}{3}\right)$. 5

5. (a) Check the continuity of a function f at x = 0 : 5

$$f(x) = \begin{cases} \frac{2|x|}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$$

(b) Find the Vector and Cartesian equations of the line passing through the point (1, −1, −2) and parallel to the vector $3\hat{i} - 2\hat{j} + 5\hat{k}$. 5

(c) Find the shortest distance between the lines $\vec{r} = (3\hat{i} + 4\hat{j} - 2\hat{k}) + t(-\hat{i} + 2\hat{j} + \hat{k})$ and $\vec{r} = (\hat{i} - 7\hat{j} - 2\hat{k}) + t(\hat{i} + 3\hat{j} + 2\hat{k})$. 5

(d) Find the maximum value of $5x + 2y$ subject to the constraints

$$-2x - 3y \leq -6$$

$$x - 2y \leq 2$$

$$6x + 4y \leq 24$$

$$-3x + 2y \leq 3$$

$$x \geq 0, y \geq 0$$ 5
