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MMT-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

01114 **December, 2016**

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

Note : *Question no. 1 is **compulsory**. Attempt any **three** questions from questions no. 2 to 5. Use of calculator is **not** allowed.*

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1. State giving reasons whether the following statements are *True* or *False* : *5×2=10*

(a) $f(z) = \bar{z}\ \forall\ z \in \mathbf{C}$ is nowhere differentiable.

(b) e^z, $z \in \mathbf{C}$ has periodicity $2\pi i$.

(c) The radius of convergence of the power series $\sum_{n=0}^{\infty} \{3 + (-1)^n\}(z-2)^n$ is at least 1.

(d) $\int_{|z|=3/2} \frac{1}{z^2-1}\,dz = 2\pi i$.

(e) $\operatorname{Res}_{z=0}\left[\frac{1-\cos z}{z^3}\right] = \frac{1}{2}$.

2. (a) Evaluate $\int_C \frac{1}{z}\,dz$, where C is the circle, $x = \cos t, y = \sin t, 0 \le t \le 2\pi$. 2

(b) Determine the linear fractional transformation that maps $z_1 = 0$, $z_2 = 1$, $z_3 = \infty$ onto $w_1 = -1$, $w_2 = -i$, $w_3 = 1$, respectively. 3

3. (a) Let $0 < |a| < |b|$. Evaluate

$$\int_{|z|=r} \frac{dz}{(z-a)(z-b)}$$

where $|a| < r < |b|$. 2

(b) Find the Laurent series of $f(z) = \frac{1}{z^3 - z^4}$ about $z = 0$. 3

4. (a) Verify that the function $u(x, y) = e^x \cos y + x$ is harmonic in the entire complex plane and find the harmonic conjugate function of u. 3

(b) Suppose f is analytic in the annulus $1 < |z| < 2$ where $|f| < 1$ on $|z| = 1$ and $|f| < 4$ on $|z| = 2$. Prove that

$|f(z)| < |z|^2$ throughout the annulus. 2

5. Show that

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^{n+1}} = \pi \frac{1.3.5 \ldots (2n-1)}{2.4.6 \ldots (2n)}$$

for all $n \in \mathbf{N}$. 5