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BME-015

B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

00202

Term-End Examination

December, 2016

BME-015 : ENGINEERING MATHEMATICS – II

Time : 3 hours *Maximum Marks : 70*

Note : *Answer any **ten** questions. All questions carry equal marks. Use of scientific calculator is permitted.*

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1. Test the convergence of the series

$$\sum_{n=1}^{\infty} \left(\sqrt{n+1} - \sqrt{n}\right).$$ 7

2. Taking $M_n = \frac{1}{n(n+1)}$, prove that

$$\sum_{n=1}^{\infty} \frac{\sin (x + nx)}{n(n+1)},$$

is uniformly convergent for all real values of x. 7

3. Find the Fourier series generated by the periodic function $|x|$ for period 2π. Also compute the values of the series for $x = 0, 2\pi$ and -3π. 7

4. Find half-range sine series for the function

$$f(x) = \begin{cases} 1 & \text{for } 0 < x < \pi/2 \\ 0 & \text{for } \pi/2 < x < \pi \end{cases}$$ 7

5. If n is a positive integer, prove that

$$(1 + i\sqrt{3})^n - (1 - i\sqrt{3})^n = 2^{n+1} \cos \frac{n\pi}{3}.$$

Hence find the value, when $n = 15$. 7

6. Determine the analytic function $w = u + iv$ if

$$u - v = (x - y)(x^2 + 4xy + y^2)$$

and express w in terms of z. 7

7. Obtain the first four terms of the Laurent's series expansion of $\dfrac{e^z}{z(z^2 + 1)}$, for $0 < |z| < 1$. 7

8. If $x + \dfrac{1}{x} = 2 \cos \theta$, $y + \dfrac{1}{y} = 2 \cos \phi$, prove that one of the values of $\dfrac{x^m}{y^n} + \dfrac{y^n}{x^m}$ is $2 \cos (m\theta - n\phi)$. 7

9. If $\alpha + i\beta = \dfrac{1}{a + ib}$, prove that

$$(\alpha^2 + \beta^2)(a^2 + b^2) = 1.$$ 7

10. Find the value of

$$\int_{c:|z|=1} \frac{e^{2z}}{(z+1)^2} dz.$$ 7

11. Evaluate

$$\int_{1-i}^{2+i} (2x + 2iy + 3)\, dz$$

along

(a) the path $x = t + 1,\ y = 2t^2 - 1$,

(b) the straight line joining $1 - i$ and $2 + i$. 7

12. Find the bilinear transformation whose fixed points are 2 and 3. 7

13. Test the convergence of the series

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots \infty.$$ 7

14. A horizontal tube is in rotation about a vertical axis with constant angular velocity ω. A sphere inside the tube slides along it without friction, so that the governing equation is $\frac{d^2r}{dt^2} = \omega^2 r$. Find the motion of the sphere if at initial instant it lies on the axis of rotation, i.e., $r(0) = 0$ and has velocity one unit along the tube, i.e., $\dot{r}(0) = 1$. Thus solve $\frac{d^2r}{dt^2} = \omega^2 r$ with $r(0) = 0$, $\dot{r}(0) = 1$. 7

15. Solve any ***one*** of the following equations : 7

(a) $(D^2 + 3DD_1 + D_1^2)\, z = e^{x + 2y}$

(b) Find the deflection $u(x, t)$ satisfying IBVP

$$u_{tt} - u_{xx} = 0 \qquad 0 < x < 1,\ t > 0$$

$$u(0, t) = 0 = u(1, t),\ t \geq 0$$

$$u(x, 0) = 0,\ 0 \leq x \leq 1$$

and $$u_t(x, 0) = \begin{cases} x & \text{for } 0 \leq x < \frac{1}{2} \\ 1 - x & \text{for } \frac{1}{2} \leq x \leq 1. \end{cases}$$