

**MCA (Revised)**  
**Term-End Examination**

07545

December, 2016

**MCS-033 : ADVANCED DISCRETE MATHEMATICS***Time : 2 hours**Maximum Marks : 50*

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*Note : Question no. 1 is compulsory. Attempt any three questions from the rest.*

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1. (a) Using induction, show that

$$T_n = 2^n - 1, \text{ where } n \geq 1,$$

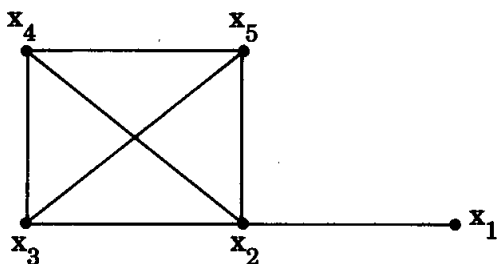
$$\text{where } T_n = 2T_{n-1} + 1. \quad 5$$

(b) Prove that the complement of  $\overline{G}$  is  $G$ . 5

(c) Draw at least 3 non-isomorphic graphs on 4 vertices. 5

(d) Prove that  $a_n = \frac{3n}{2} - 2$  is a solution to the recurrence  $a_n = 2a_{n/2} + 2$ , where  $n$  is a power of 2 and  $a_2 = 1$ . 5

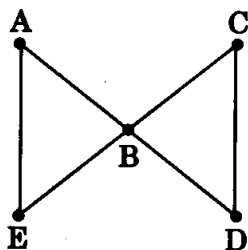
2. (a) Find the number of bijections on a set of  $n$  elements, where  $n \geq 1$ . 5
- (b) Consider the graph on 5 vertices and 7 edges given in the figure. Find  $x_1$  to  $x_5$  walks of length 8 and length 4 respectively. 5



3. (a) If  $G$  is a graph with  $n$  vertices and  $k$  components, then prove that  $G$  can have at least  $n - k$  edges and at most  $\left\{ \frac{(n - k)(n - k + 1)}{2} \right\}$  edges. 5
- (b) Solve the third order recurrence  $U_n - 9U_{n-1} + 26U_{n-2} - 24U_{n-3} = 0$ , where  $n \geq 3$ , with the initial conditions  $U_0 = 6, U_1 = 17$  and  $U_2 = 53$ . 5
4. (a) Solve the recurrence  $d_k = k d_{k-1} + (-1)^k$  if  $k \geq 2$  with  $d_1 = 0$ . 5
- (b) Find  $\lambda(G)$ , where  $G$  is the Petersen graph. 5

5. (a) Draw three spanning trees of the following graph :

3



- (b) What is the difference between an Eulerian graph and an Eulerian circuit ?

2

- (c) Construct a graph with chromatic number 5.

3

- (d) Solve the recurrence relation  $a_n = a_{n/2} + 1$  for  $n = 2^k$ , where  $k \geq 1$ ,  $a_1 = 0$ .

2

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