MCA (Revised)
Term-End Examination
ISSE December, 2016

## MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours

Maximum Marks : 100

Note: Question no. 1 is compulsory. Attempt any three from the remaining questions.

1. (a) Use Mathematical Induction to prove that

$$
\sum_{i=1}^{n} i=\frac{n(n+1)}{2}
$$

(b) For a problem P , two algorithms $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ have time complexities $T_{1}(n)=5 n^{2}$ and $T_{2}(n)=100 \mathrm{n} \log \mathrm{n}$. Find the range for n , the size of instance of the given problem $P$, for which $A_{1}$ is more efficient than $A_{2}$.
(c) Define the big theta notation. Show that $n^{2}+3 \log n=\theta\left(n^{2}\right)$.
(d) (i) Explain the bottom-up build heap procedure.
(ii) Illustrate heapsort algorithm on the sequence $\langle 10,5,12,25,2,8,13,7\rangle .10$
(e) Solve the following recurrence equations: 10
(i) $\mathrm{T}(\mathrm{n})=2 \mathrm{~T}(\mathrm{n} / 2)+\mathrm{O}(\mathrm{n})$
(ii) $T(n)=T(n-1)+O(n)$
(f) Write a Regular expression to generate strings of even length over the alphabet $\Sigma=\{a, b\}$.
2. (a) Give a divide and conquer algorithm to find the $i^{\text {th }}$ smallest in an unsorted list of $n$ integers. Show that the algorithm works in $O(n)$ time.
(b) Write a recursive function to calculate the sum of all elements in an integer array.
(c) Explain any two applications of DFS traversal algorithm.
3. (a) Given the currency coins of denomination 1,4 and 6. Design a dynamic programming algorithm to obtain minimum number of coins for a given amount.
(b) Using Prim's algorithm, find a Minimal Spanning tree for the graph given below : 10

4. (a) (i) Write a context-free grammar to generate all palindromes of even length over the alphabet $\Sigma=\{\mathrm{a}, \mathrm{b}\}$.
(ii) Derive the parse tree and derivation for the string aabbaa.
(b) (i) Explain the algorithm to find the Strongly Connected Component in an undirected graph.
(ii) Find the Strongly Connected Components in the following graph :

5. (a) Explain the following : ..... 10
(i) Undecidable problems(ii) Turing machines
(b) Define the Class P, NP and NP-complete problems.5
(c) Write a Turing machine to recognize the language of all strings of even length over the alphabet $\{\mathrm{a}, \mathrm{b}\}$.5

