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BCS-012

BACHELOR OF COMPUTER APPLICATIONS (BCA) (Revised)

Term-End Examination

08955

December, 2016

BCS-012 : BASIC MATHEMATICS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number 1 is **compulsory**. Attempt any **three** questions from the remaining four questions.*

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1. (a) Evaluate the determinant

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix},$$ where ω is a cube root of unity. *5*

(b) Using determinant, find the area of the triangle whose vertices are $(-3, 5)$, $(3, -6)$ and $(7, 2)$. *5*

(c) Use the principle of mathematical induction to show that $2 + 2^2 + \ldots + 2^n = 2^{n+1} - 2$ for every natural number n. *5*

(d) Find the sum of all integers between 100 and 1000 which are divisible by 9. *5*

(e) Check the continuity of the function f(x) at x = 0 : 5

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

(f) If $y = \frac{ln\, x}{x}$, show that $\frac{d^2y}{dx^2} = \frac{2\, ln\, x - 3}{x^3}$. 5

(g) If the mid-points of the consecutive sides of a quadrilateral are joined, then show (by using vectors) that they form a parallelogram. 5

(h) Find the scalar component of projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ on the vector $\vec{b} = 2\hat{i} - 2\hat{j} - \hat{k}$. 5

2. (a) Solve the following system of linear equations using Cramer's rule : 5

$x + 2y - z = -1,$

$3x + 8y + 2z = 28,$

$4x + 9y + z = 14.$

(b) Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$.

Show that $f(A) = O_{2\times2}$. Hence find A^5. 10

(c) Determine the rank of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \\ 5 & 3 & 14 & 4 \end{bmatrix}.$$ 5

3. (a) The common ratio of a G.P. is $-4/5$ and the sum to infinity is $80/9$. Find the first term of the G.P. 5

(b) If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then show that $a = 1$, $b = 0$. 5

(c) Solve the equation $8x^3 - 14x^2 + 7x - 1 = 0$, the roots being in G.P. 5

(d) Find the solution set for the inequality $15x^2 + 4x - 4 \geq 0$. 5

4. (a) If a mothball evaporates at a rate proportional to its surface area $4\pi r^2$, show that its radius decreases at a constant rate. 5

(b) Find the absolute maximum and minimum of the function $f(x) = \frac{x^3}{x+2}$ on the interval $[-1, 1]$. 5

(c) Evaluate the integral

$$I = \int \frac{dx}{1 + 3e^x + 2e^{2x}}.$$ 5

(d) Find the length of the curve

$y = 2x + 3$ from $(1, 5)$ to $(2, 7)$. 5

5\. (a) Find the value of λ for which the vectors

$\vec{a} = \hat{i} - 4\hat{j} + \hat{k}$, $\vec{b} = \lambda\hat{i} - 2\hat{j} + \hat{k}$ and

$\vec{c} = 2\hat{i} + 3\hat{j} + 3\hat{k}$ are coplanar. *5*

(b) Find the equations of the line (both Vector and Cartesian) passing through the point (1, −1, −2) and parallel to the vector $3\hat{i} - 2\hat{j} + 5\hat{k}$. *5*

(c) A manufacturer makes two types of furniture, chairs and tables. Both the products are processed on three machines A_1, A_2 and A_3. Machine A_1 requires 3 hours for a chair and 3 hours for a table, machine A_2 requires 5 hours for a chair and 2 hours for a table and machine A_3 requires 2 hours for a chair and 6 hours for a table. The maximum time available on machines A_1, A_2 and A_3 is 36 hours, 50 hours and 60 hours respectively. Profits are ₹ 20 per chair and ₹ 30 per table. Formulate the above as a linear programming problem to maximize the profit and solve it. *10*
