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MST-003

POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

Term-End Examination

December, 2015

MST-003 : PROBABILITY THEORY

Time : 3 hours *Maximum Marks : 50*

Note :

(i) *Attempt **all** questions.*

(ii) *Questions no. 2 to 5 have internal choices.*

(iii) *Use of scientific calculator is allowed.*

(iv) *Use of Formulae and Table Booklet for PGDAST is allowed.*

(v) *Symbols have their usual meanings.*

1. Which of the following statements are *True* or *False* ? Give reasons in support of your answer.

5×2=10

(a) Classical definition of probability fails if the cases are not equally likely.

(b) If a coin is tossed twice, then the sample space is {HH, HT, TT}.

(c) If a fair die is thrown once, then the probability of getting an even number is greater than that of getting an odd number.

(d) If X follows Poisson distribution, then mean of X > variance of X.

(e) If $X \sim N(\mu, \sigma^2)$ and $Z = \frac{X - \mu}{\sigma}$, then $Z \sim N(0, \sqrt{2})$.

2. (a) Find the probability of event A, if

(i) odds in favour of event A are 4 : 3,

(ii) odds against event A are 5 : 8. *2*

(b) If $P(A) = \frac{3}{5}$, then find

(i) odds in favour of A,

(ii) odds against the occurrence of event A. *2*

(c) If A, B and C are any three events, write down the expressions for the following in terms of set theory :

(i) only A occurs,

(ii) A and B occur but C does not occur,

(iii) A, B and C, all the three occur,

(iv) at least two occur,

(v) exactly two do not occur,

(vi) none occurs. *6*

OR

(a) A die is rolled. If the outcome is a number greater than 3, what is the probability that it is a prime number ? *4*

(b) A couple has 2 children. What is the probability that both the children are boys, if it is known that

(i) younger child is a boy,

(ii) elder child is a boy,

(iii) at least one of them is a boy ? *6*

3. (a) 2 bad articles are mixed with 5 good ones. Find the probability distribution of the number of bad articles, if 2 articles are drawn at random. *5*

(b) The following table represents the joint probability distribution of the discrete random variable (X, Y) :

X \ Y	1	2
1	0·1	0·2
2	0·1	0·3
3	0·2	0·1

Find

(i) The marginal distributions,

(ii) The conditional distribution of X given $Y = 1$,

(iii) $P[X + Y < 4]$. *5*

OR

(a) Let the joint density function of a two-dimensional random variable (X, Y) be given by

$$f(x,y)=\begin{cases} x+y, & 0 \le x < 1 \text{ and } 0 \le y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the conditional density function of Y given X. 5

(b) A player tosses two unbiased coins. He wins ₹ 5 if 2 heads appear, ₹ 2 if one head appears and ₹ 1 if no head appears. Find the expected value of the amount won by him. 5

4. (a) If $X \sim B(n, p)$, find p if n = 6 and $9P[X = 4] = P[X = 2]$. 5

(b) If the probability that an individual suffers a bad reaction from an injection of a given serum is 0·001, determine the probability that out of 1500 individuals, exactly 3 individuals suffer from a bad reaction. 5

OR

(a) A jury of 5 members is drawn at random from a voters' list of 100 persons, out of which 60 are non-graduates and 40 are graduates. What is the probability that the jury will consist of 3 graduates ? 5

(b) Comment on the following :

The mean and variance of geometric distribution are 4 and 3, respectively. 3

(c) Find the probability that the third head turns up in the 5^{th} toss of an unbiased coin. 2

5. (a) Write any five chief characteristics of the normal distribution. 5

(b) If the random variable X is normally distributed with mean 80 and standard deviation 5, then find the P[60·5 < X < 90]. 5

OR

(a) Obtain the value of $k > 0$ for which the function given by

$$f(x) = 2e^{-kx},\ x \geq 0,$$

follows an exponential distribution. Hence, find its mean and variance. 5

(b) Obtain the mean and variance for the beta distribution whose density function is given by

$$f(x) = \frac{60x^2}{(1+x)^7},\ 0 < x < \infty\ .$$ 5