No. of Printed Pages : 3

MMTE-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00954

Term-End Examination

December, 2015

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Answer any* ***five*** *questions from questions no. 1 to 6. Calculators are* ***not*** *allowed.*

1. (a) Prove that, in a linear code, the minimum distance is the same as the minimum weight. *3*

(b) State and prove the sphere packing bound. *4*

(c) Find all the primitive elements in $\mathbf{F}_{11}$. *3*

2. (a) Find all the code-words of the code $\mathcal{C}$ with generator matrix

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

How many errors can $\mathcal{C}$ detect ? How many can it correct ? *6*

(b) Construct a field with 8 elements. *4*

3. (a) Let C be [15, 7] narrow-sense binary BCH code of designed distance $\delta = 5$, which has defining set

$$T = \{1, 2, 3, 4, 6, 8, 9, 12\}.$$

Let $\alpha^4 = 1 + \alpha$, where α is primitive 15^{th} root of unity, and generator polynomial of C is

$$g(x) = 1 + x^4 + x^6 + x^7 + x^8.$$

If $y(x) = 1 + x + x^5 + x^6 + x^9 + x^{10}$ is received, find the transmitted code word. *5*

(b) Define cyclic code and give an example. *3*

(c) Prove that a BCH code of designed distance δ has minimum weight at least δ. *2*

4. (a) Let C be a cyclic code over $\mathbf{F}_q$ with generating idempotent e(x). Prove that the generator polynomial of C is

$g(x) = \gcd(e(x), x^n - 1)$ computed in $\mathbf{F}_q[x]$. *5*

(b) Let C be any self-dual [12, 6, 6] ternary code. Prove that the weight enumerator of C is

$$W_C(x, y) = y^{12} + 264x^6y^6 + 440x^9y^3 + 24x^{12}. \quad 5$$

5. (a) Construct the generating idempotents of the duadic codes of length 11 over $\mathbf{F}_3$. *6*

(b) Let C be the $\mathbf{Z}_4$ – linear code of length 3 with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}.$$

(i) List the 16 code-words in C. *2*

(ii) List the 16 code-words in the Gray image of C. *2*

6. (a) Define a convolutional code and give an example. *3*

(b) If a polynomial generator matrix of an (n, k) convolutional code C is basic and reduced, prove that G is canonical. *3*

(c) Write the Message Passing Decoding Algorithm. *4*