MMTE-001

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

December, 2015

00724

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*
(Weightage : 50%)

Note : *Question no. 1 is* ***compulsory****. Answer any* ***four*** *questions out of the remaining six numbered 2 to 7. Calculating devices are* ***not*** *allowed.*

1. State whether the following statements are *true* or *false*, with a brief justification : *5×2=10*

(a) The cycle C_n is bipartite for every integer $n \geq 3$.

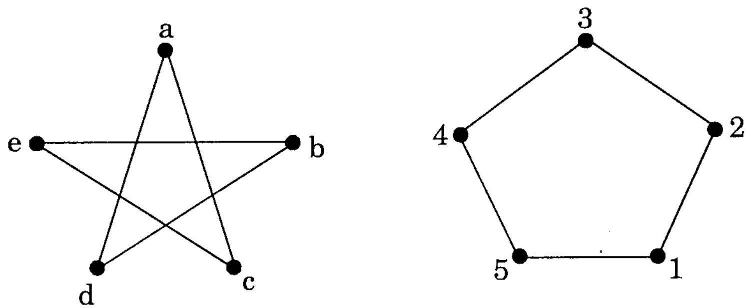
(b) The complement of a disconnected graph is always connected.

(c) Every Eulerian graph is Hamiltonian.

(d) The chromatic number of $K_{m, n}$ is $\min(m, n)$.

(e) Every simple graph is 4-colourable.

2. (a) Check whether the graphs in the figure below are isomorphic or not. Justify your answer. 3



(b) If $\delta(G) \geq 2$ for a graph G, then prove that G contains a cycle. 3

(c) Define $\alpha'(G)$ and $\beta'(G)$. Prove that $\alpha'(G) + \beta'(G) = n(G)$. 4

3. (a) If G is a simple graph which is self-complementary, then prove that $\text{diam}(G) = 3$. 5

(b) Prove that a simple graph is connected, if and only if it has a spanning tree. Further, show that a simple graph is a tree, if and only if it has exactly one spanning tree. 5

4. (a) Let $A = (A_i, \dots, A_m)$ be a collection of subsets of a set Y. A System of Distinct Representatives (SDR) for A is a set of distinct elements $a_i, \dots, a_m \in Y$ such that $a_i \in A_i$ for $i \in \{l, \dots, m\}$. Prove that A has an SDR, if and only if

$$\left| \bigcup_{i \in S} A_i \right| \geq |S| \ \forall\, S \subseteq \{1, 2, \dots, m\}.$$ 6

(b) Prove that the minimum number of edges in a K-connected graph on n vertices is $\left\lceil \frac{Kn}{2} \right\rceil$. 4

5. (a) If f is a feasible flow and [S, T] is a source-sink cut, then prove that the net flow out of S equals the net flow into T. 3

(b) Show that in a k-critical graph, G, $\delta(G) \geq k - 1$. 3

(c) If G is a simple graph on n vertices, where $n \geq 3$, and $\delta(G) \geq \frac{n}{2}$, then prove that G is Hamiltonian. 4

6. (a) Draw a graph, G, with a vertex v so that $\chi(G - v) < \chi(G)$ and $\chi(\overline{G} - v) < \chi(\overline{G})$. 3

(b) Prove that $K_{3,3}$ is non-planar. 3

(c) Show that the number of vertices of degree 1 in any tree is $1 + \frac{1}{2} \sum_{v \in V} \left| d(v) - 2 \right|$, where V is the set of vertices of the tree. *4*

7. (a) If h is a connected planar, simple graph with n vertices and e edges, $n \geq 3$, with girth 5, prove that $3e \leq 5(n-2)$. Deduce that the Petersen Graph is not planar. *4*

(b) Show that the sequence $\{1, 1, 2, 2, \ldots, k, k\}$ is graphical, for every $k \geq 1$. *3*

(c) Let xy be an edge in a simple graph, G. Prove that

$$\kappa(G - xy) \geq \kappa(G) - 1.$$ *3*
