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MMT-009

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

December, 2015

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Answer any **five** questions. Use of calculators is **not** allowed.*

1. (a) Explain, with examples, the difference between

(i) Static and Dynamic Models,

(ii) Deterministic and Stochastic Models. *2*

(b) Compare the risk of two securities 1 and 2 whose return distributions are given below : *3*

Possible Rate of Return for Security		Associated Probability
1	2	$p_{1j} = p_{2j}$
0·19	0·09	0·13
0·17	0·16	0·15
0·11	0·18	0·42
0·10	0·11	0·30

2. Consider the discrete time population model given by

$$N_{t+1} = \frac{rN_t}{1+\left(\frac{N_t}{K}\right)^b}, \text{ for a population } N_t,$$

where K is the carrying capacity of the population, r is the intrinsic growth rate and b is a positive parameter. Determine the non-negative steady-state and discuss the linear stability of the model for $0 < r < 1$. Also find the first bifurcation value of the parameter r. 5

3. Do the stability analysis of the following pre-predator model formulated to study the effect of toxicants on the competing species :

$$\frac{dN_1}{dt} = r_1N_1 - \alpha_1N_1N_2 - d_1C_0N_1$$

$$\frac{dN_2}{dt} = r_2N_2 - \alpha_2N_1N_2$$

$$\frac{dC_0}{dt} = k_1P - g_1C_0 - m_1C_0$$

$$\frac{dP}{dt} = Q - hP - kPN_1 + gC_0N_1$$

under the conditions

$N_1(0) = N_{10}$, $N_2(0) = N_{20}$, $C_0(0) = 0$, $P(0) = P_0 > 0$,

where $N_1(t)$ = Density of prey population,

$N_2(t)$ = Density of predator population,

$C_0(t)$ = Concentration of the toxicant in the individual of the prey population,

P = Constant environmental toxicant concentration,

Q = The exogenous rate of input of toxicant into the environment,

r_1, α_1, d_1, r_2, α_2, k_1, g_1, m_1, h, k and g are all positive constants. *5*

4. A company has three factories F_1, F_2 and F_3 that supply to three markets M_1, M_2 and M_3. The transportation costs from each factory to each market are given in the table. Capacities a_i's of the factories and market requirements b_j's are shown below. Find the minimum transportation cost. *5*

	M_1	M_2	M_3	a_i
F_1	2	1	3	20
F_2	1	2	3	30
F_3	2	1	2	10
b_j	10	10	20	40/60

5. (a) Given a set of securities with portfolio values

$$w_i's = \frac{1}{3}, \frac{2}{3}, \frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}.$$

Find a feasible set of portfolio of these securities. *2*

(b) Formulate the model for which the reproductive function of the cancer cells in the tumour surface is given by $\phi(c) = \frac{c-1}{1-2c}$, $c \neq \frac{1}{2}$ together with initial conditions $c = 20 \times 10^5$ at $t = 0$. Also find the density of the cancer cells in the tumour's surface area at $t = 45$ days. *3*

6. (a) What are residual plots and box plots ? Give an example of each. *2*

(b) Let G(t) be the amount of the glucose in the bloodstream of a patient at time t. The glucose is infused into the bloodstream at a constant rate of k gm/min. At the same time, the glucose is converted and removed from the bloodstream at a rate proportional to the amount of glucose present. If the initial concentration of glucose in the bloodstream was G_0, then find the concentration at any time t. Also find the limiting value of the concentration as $t \to \infty$. *3*