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MMT-007

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

December, 2015

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no. 1 is **compulsory**. Answer any **four** questions out of questions no. 2 to 7. Use of calculators is **not** allowed.*

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1. State whether the following statements are *true* or *false*. Justify your answers with the help of a short proof or a counter-example. *5×2=10*

(a) The differential equation $\frac{dy}{dx} = xy^2$, satisfies the Lipschitz condition on any strip $a \le x \le b, -\infty < y < \infty$.

(b) Inverse Laplace transform of $\cot^{-1} s$, is

$$\mathcal{L}^{-1}(\cot^{-1} s) = \frac{\sin t}{t}.$$

(c) The interval of absolute stability of the method

$$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$$

$$k_1 = hf(x_n, y_n), k_2 = hf(x_n + h, y_n + k_1)$$

for the solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$, is $]-2, 0[$.

(d) The explicit scheme

$$u_i^{n+1} = u_i^n + \lambda[u_{i+1}^n - 2u_i^n + u_{i-1}^n],$$

$\lambda = k/h^2$ for solving the parabolic equation $u_t = u_{xx}$ is stable for $\lambda < 1$.

(e) $x = 0$ is an irregular singular point of the differential equation

$$x\, y'' + \sin(2x)\, y' + xe^{3x}\, y = 0.$$

2. (a) Prove the recurrence relation of the Lagrange polynomials

$$(1 - x^2)\, P_n'(x) = (n + 1)\, [xP_n(x) - P_{n+1}(x)],$$

$$n = 0, 1, 2, \dots .$$ 4

(b) The heat conduction equation $u_t = u_{xx}$ is approximated by

$$u_m^{n+1} = u_m^{n-1} + 2\lambda\,(u_{m-1}^n - 2u_m^n + u_{m+1}^n), \lambda = k/h^2.$$

(i) Find the truncation error of the method.

(ii) Investigate the stability of the method. 6

3. (a) Find the series solution about x = 0, of the differential equation

$$x\,y'' + (1 - 2x)\,y' + (x - 1)\,y = 0.$$ 5

(b) Reduce the second order initial value problem

$$y'' = y' + 3$$
$$y(0) = 1$$
$$y'(0) = \sqrt{3}$$

to a system of first order initial value problems. Hence, find y(0·1), y′(0·1), using Taylor series method of second order with h = 0·1. 5

4. (a) Show that

$$J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{1}{x^2}(3 - x^2)\sin x - \frac{3}{x}\cos x\right).$$ 2

(b) Show that

$$\int_{-1}^{1} x\, P_n(x)\, P_{n-1}(x)\, dx = \frac{2n}{4n^2 - 1},$$

where $P_n(x)$ is n^{th} degree Legendre polynomial. 4

(c) A region R is divided into rectangular elements, whose sides are parallel to x and y axes. An element has corners $P(x_i, y_i)$, $Q(x_j, y_i)$, $R(x_j, y_m)$ and $S(x_i, y_m)$. Derive an interpolating polynomial $u(x, y)$ that can be used in the element PQRS. 4

5. (a) Using the Fourier transform, solve the initial boundary value problem

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \; -\infty < x < \infty, \; t > 0,$$

$$u(x, 0) = f(x),$$

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0.$$ 5

(b) Solve the boundary value problem using the finite difference method with second order approximations with $h = 1/2$: 5

$$y'' + 5y' + 4y = 2$$

$$y(0) + y'(0) = 1, \; y(1) = 4.$$

6. (a) Using Green's function method, solve the boundary value problem

$$y'' + y = x^2, \ y(0) = 0 = y\left(\frac{\pi}{2}\right).$$ 5

(b) Determine the interval of absolute stability for the method 5

$$y_{i+1} = \frac{9}{8}y_i - \frac{1}{8}y_{i-2} + \frac{3h}{8}(y'_{i+1} + 2y'_i - y'_{i-1}),$$

when applied to the test equation

$y' = \lambda y, \ \lambda < 0.$

7. (a) Using Laplace transforms, solve the initial boundary value problem

$$u_{tt} = u_{xx}, \ 0 < x < \pi, \ t > 0,$$

$$u(x, 0) = 0, \ \frac{\partial u}{\partial t} = \sin x \text{ at } t = 0,$$

$$u(0, t) = 0, \ u(\pi, t) = 0, \ t > 0.$$ 4

(b) Using the Crank-Nicolson method, integrate up to 2 time levels for the solution of the initial boundary value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \ 0 \le x \le 1,$$

$$u(x, 0) = \sin(2\pi x),$$

$$u(0, t) = 0 = u(1, t)$$

with $h = 1/3$ and $\lambda = 1/6$. 6
