

**M.Sc. (MATHEMATICS WITH APPLICATIONS
IN COMPUTER SCIENCE)**

M.Sc. (MACS)

Term-End Examination

December, 2015

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ hours

Maximum Marks : 25

Note : *Question no. 1 is compulsory. Attempt any three questions from questions no. 2 to 5. Use of calculator is **not** allowed.*

1. State giving reasons whether the following statements are *true* or *false* : $5 \times 2 = 10$

(a) The union of two open connected sets is an open connected set.

(b) $\int_C \frac{7}{2z-3} dz = 0$, where C is the unit circle centred at origin.

(c) $f(z) = e^{i|z|}$ is an entire function.

(d) The power series $\sum_{n=1}^{\infty} \frac{1}{n^2} z^n$ converges on the closed unit disc $D = \{ z : |z| \leq 1 \}$.

(e) Mapping $f(z) = e^z \sin z$ is conformal at all the points of the complex plane.

2. (a) Show that the function

$$f(z) = \frac{\bar{z}}{|z|}, \quad z \neq 0$$

$$f(0) = 0$$

is not continuous at $z = 0$.

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(b) Show that $u(x, y) = y + e^x \cos y$ is a harmonic function. Find its harmonic conjugate and the analytic function $f(z)$ whose real part is u .

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3. (a) State the maximum modulus principle and use it to find the maximum modulus of $f(z) = 2z + 5i$ on the closed circular region defined by $|z| \leq 2$.

$2\frac{1}{2}$

(b) Evaluate

$$\int_C \frac{ze^z}{2z^2 - 3z - 2} dz,$$

where C is $\{ z : |z - 1| = 2 \}$.

$2\frac{1}{2}$

4. (a) Locate the singularities of the following function and determine their type :

2

$$f(z) = (z-1)^2 \sin \left(\frac{1}{z-1} \right).$$

- (b) If $f(z)$ is an entire function such that $|f(z)| \leq 5|z|$ for all z , then show that $f(z) = az$ for some constant a .

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5. Evaluate $\int_0^{\infty} \frac{1}{(x^2+4)^2} dx$.

5
