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MMT-003

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

December, 2015

MMT-003 : ALGEBRA

Time : 2 hours *Maximum Marks : 50*

(Weightage : 70%)

Note : *Question no. 1 is* ***compulsory****. Answer any* ***four*** *questions from questions no. 2 to 6. Calculators are* ***not*** *allowed.*

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1. State, with reasons, whether the following statements are *True* or *False* : *5×2=10*

(a) Any subgroup $H \subset G$ of a group G is the kernel of a suitable group homomorphism $\phi : G \to G_1$.

(b) If $m > 1$, $n > 1$ are natural numbers with $m > n$, there is a group G of order m and a set S with n elements such that G operates transitively on S.

(c) For every $n \in \mathbf{N}$, $n > 2$, there are infinitely many irreducible polynomials of degree n over Q.

(d) Let $R_1, R_2 : G \to GL_n(\mathbf{R})$ be two irreducible representations of the finite group G. Then, their characters χ_{R_1} and χ_{R_2} are orthogonal only if R_1 and R_2 are inequivalent.

(e) The fields $\mathbf{Q}(\pi)$ and $\mathbf{Q}(\pi^2)$ are isomorphic.

2. (a) Solve the set of congruences

$$x \equiv 2 \pmod 5$$

$$2x \equiv 3 \pmod 7$$

$$x \equiv 4 \pmod{11}$$

simultaneously. 5

(b) Show that $L = \{ x\, y^{2n} \mid n \geq 0 \}$ is a regular language. 2

(c) Let α be the real cube root of 2 and let β be another (complex) root of $X^3 - 2 = 0$. Show that the fields $\mathbf{Q}(\alpha)$ and $\mathbf{Q}(\beta)$ are isomorphic. 3

3. (a) Let $\sigma = (1\ 2\ 3\ 4)\ (5\ 6)\ (7\ 8\ 9)\ (10\ 11) \in A_{11}$. Check whether the conjugacy class of σ in S_{11} splits into two conjugacy classes in A_{11} or not. 3

(b) Calculate the Legendre Symbol $\left(\frac{15}{71}\right)$. 3

(c) (i) Does $X^p - X$ have a multiple root over $\mathbf{Z}_p$?

(ii) Let p be an odd prime and $f(X) = X^p + 1$. Then, show that all roots of f over $\mathbf{Z}_p$ are multiple roots. 4

4. (a) Check whether there is a group of order 12 with class equation $1 + 2 + 4 + 5$. 2

(b) Prove that the subgroup SO_2 of SU_2 is conjugate to the subgroup T of diagonal vectors. 4

(c) Show that $\cos \frac{\pi}{8}$ is an algebraic number. 4

5. (a) Let G be an abelian group of order n and suppose p is a prime such that $p^k \mid n$ and $p^{k+1} \nmid n$. Let H be the Sylow p-group of G and $m = \frac{n}{p^k}$. Show that, for any $a \in G$, $a^m \in H$. 4

(b) Show that any field extension of degree 2 is normal. 3

(c) If G is a group of order n, then show that the number of inequivalent irreducible representations of G is at most n. 3

6. (a) Find the elementary divisors of the group

$\mathbf{Z}_4 \times \mathbf{Z}_6 \times \mathbf{Z}_{21} \times \mathbf{Z}_{35}$. 2

(b) Determine the last row of the following character table of a group G of order 12 which has 4 conjugacy classes : 5

	1	3	4	4
	x_1	x_2	x_3	x_4
χ_1	1	1	1	1
χ_2	1	1	ω^2	ω
χ_3	1	1	ω	ω^2
χ_4	-	-	-	-

(c) Show that $\mathbf{Q}(\sqrt{3}+\sqrt{7}) = \mathbf{Q}(\sqrt{3}, \sqrt{7})$. 3