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MMT-002

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

December, 2015

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Question no.* **5** *is* ***compulsory****. Answer any* ***three*** *questions from questions no. 1 to 4. Use of calculator is* ***not*** *allowed.*

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1. (a) Let V be the vector space of polynomials of degree at most 2 with real coefficients. Let $T : V \to \mathbf{R}^3$ be a linear transformation defined by $T\,f(x) = \begin{bmatrix} f(-1) \\ f(0) \\ f(1) \end{bmatrix}$.

Find the matrix of T relative to the basis $\{1, x, x^2\}$ of V and the standard basis of $\mathbf{R}^3$. *2*

(b) Find the quadratic polynomial which best fits the points (–1, 0), (0, 2), (1, 1) and (2, –1). *3*

2. (a) Find the Jordan form of A, where 3

$$A = \begin{bmatrix} 1 & 1 & -2 & 0 \\ 2 & 1 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & -1 & 2 & 1 \end{bmatrix}.$$

You are given that $(x-1)^4$ is a characteristic polynomial of A.

(b) Consider the predator-prey system given by

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} 0\cdot 6 & 0\cdot 5 \\ -0\cdot 16 & 1\cdot 2 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \end{bmatrix}.$$

What is the long term behaviour of $\begin{bmatrix} x_k \\ y_k \end{bmatrix}$? 2

3. Find the singular value decomposition for the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$. 5

4. (a) Solve the system of differential equations

$$\frac{dy(t)}{dt} = Ay(t) \text{ with } y(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

where $A = \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$. 3

(b) Check whether the matrix

$$\begin{bmatrix} 7 & 0 & 24 \\ 4 & 1 & 16 \\ -2 & 0 & -7 \end{bmatrix}$$ is diagonalisable. 2

5. Which of the following statements are *true* and which are *false* ? Justify your answer. *5×2=10*

(a) For any normal operator T,

$$\ker T = \ker T^*.$$

(b) A matrix has a unique generalized inverse.

(c) Up to similarity, there is a unique matrix with characteristic polynomial $(x - 1)^4$ and minimal polynomial $(x - 1)^2$.

(d) If A is an $n \times n$ matrix, then $\det (e^A) = e^{\det (A)}$.

(e) Product of two unitary matrices need not be unitary.