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No. of Printed Pages : 4

BICE-027

B.Tech. – VIEP – MECHANICAL ENGINEERING / B.Tech. CIVIL ENGINEERING (BTMEVI / BTCLEVI)

Term-End Examination

December, 2015

BICE-027 : MATHEMATICS-III

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any* ***ten*** *questions. All questions carry equal marks. Use of scientific calculator is permitted.*

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1. Expand f(x) = x as a half range cosine series in $0 < x < 2$. 7

2. Find the Fourier series to represent the function f(x) given by

$$f(x) = \begin{cases} -k, & \text{for} \quad -\pi < x < 0 \\ k, & \text{for} \quad 0 < x < \pi. \end{cases}$$ 7

3. Obtain the Fourier series for the function $f(x) = \frac{1}{4}(\pi - x)^2$ in the interval $0 \le x \le 2\pi$ and hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \frac{\pi^2}{6}$. 5+2=7

4. Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \leq x \leq \pi$ and also, deduce that

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \ldots\ldots = \frac{\pi - 2}{4}.$$ 7

5. Solve the partial differential equation $(y^2 + z^2)\, p - xyq = -zx.$ 7

6. Solve :

$$\frac{\partial^3 z}{\partial x^3} - 7 \frac{\partial^3 z}{\partial x\, \partial y^2} - 6 \frac{\partial^3 z}{\partial y^3} = \sin(x + 2y) + e^{3x+y}.$$ 7

7. Find the Fourier cosine transform of the function

$$f(x) = \begin{cases} x, & 0 < x < \frac{1}{2} \\ 1 - x, & \frac{1}{2} < x < 1 \\ 0 & x > 1 \end{cases}$$ 7

8. Find the Fourier transform of the function $F(x) = e^{-|x|}$. Also, find the Fourier transform by using change of scale property of the function $e^{-a|x|}$. 5+2=7

9. Use the method of separation of variables to solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$. 7

10. Solve the following equation by the method of separation of variables $\frac{\partial^2 u}{\partial x\, \partial t} = e^{-t} \cos x$ given that, $u = 0$ when $t = 0$ and $\frac{\partial u}{\partial t} = 0$, when $x = 0$. 7

11. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = A \sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by

$$y(x, t) = A \sin\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi x t}{l}\right).$$ 7

12. Find the solution of the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$, given that

(i) $u = 0$ when $x = 0$ and $x = l$ for all t

(ii) $u = 3 \sin \frac{\pi x}{l}$ when $t = 0$ for all x, $0 < x < l$. 7

13. The ends A and B of a rod of length 20 cm are at temperatures 30°C and 80°C until steady state prevails. Then the temperature of the rod ends are changed to 40°C and 60°C respectively. Find the temperature distribution function u (x, t). The specific heat, density and the thermal conductivity of the material of the rod are such that the combination $\frac{k}{l\sigma} = c^2 = 1$. 7

14. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, with the rectangle $0 \le x \le a$, $0 \le y \le b$, given that

$u(x, b) = u(0, y) = u(a, y) = 0$ and

$u(x, 0) = x(a - x)$. 7

15. Neglecting R and G, find the e.m.f. $v(x, t)$ in a line of length l, t seconds after the ends were suddenly grounded, given that $i(x, 0) = i_0$ and $v(x, 0) = e_1 \sin \frac{\pi x}{l} + e_5 \sin \frac{5\pi x}{l}$. 7