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BME-001

B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination

December, 2015

00821

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Marks are shown against each question. Use of calculator is allowed.*

1. Answer any ***five*** of the following : *5×4=20*

(a) State whether the following are *True* or *False* :

(i) $-1 \in (-\infty, 2)$ True/False

(ii) $0 \in [1, \infty]$ True/False

(b) Evaluate the limits

(i) $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$

(ii) $\lim\limits_{x \to 0} x \cot 4x$

(c) Let $y = x^n$ (n is a positive integer).

Prove that $\frac{dy}{dx} = nx^{n-1}$.

(d) Find the constant λ, so that the vectors

$a = 2\hat{i} - \hat{j} + \hat{k}, b = \hat{i} + 2\hat{j} - 3\hat{k}$,

$c = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar.

(e) Find $A^2 - 3A$, if

$$A = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{vmatrix}.$$

(f) If events A and B are independent and $P(A) = 0{\cdot}15$, $P(A \cup B) = 0{\cdot}45$, then find $P(B)$.

2. Answer any ***four*** of the following questions : *4×5=20*

(a) Differentiate

$(\sin x)^x$.

(b) Find

$$\int (x^2 + 1)^3 \, 2x \, dx.$$

(c) Show that, if $r = a \sin \omega t + b \cos \omega t$, where a and b are constants, then

$$\frac{d^2r}{dt^2} = -\omega^2 r \text{ and } r \times \frac{dr}{dt} = -\omega(a \times b).$$

(d) Define and explain Curl of a vector field.

(e) Prove that every orthogonal set of non-zero vectors is linearly independent.

(f) A problem is given to three students A, B and C, whose chances of solving the problem are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved ?

3. Attempt any ***five*** of the following questions : *5×3=15*

(a) If $A = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}, B = \begin{vmatrix} 1 & 2 \\ -1 & 0 \\ 2 & -1 \end{vmatrix}$

obtain the product AB. Also explain why BA is not defined.

(b) Solve the following system of linear equations by Cramer's Rule :

$$2x - y + 3z = 2$$

$$x + 3y - z = 11$$

$$2x - 2y + 5z = 3$$

(c) Find the eigen-values of the matrix

$$A = \begin{bmatrix} 5 & 7 & -5 \\ 0 & 4 & -1 \\ 2 & 8 & -3 \end{bmatrix}.$$

(d) Verify Cayley – Hamilton Theorem for matrix A. Also find its inverse.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

(e) Using Cayley – Hamilton Theorem, find B^3 and B^{-1} for the matrix

$$B = \begin{bmatrix} 1 & 2 \\ 8 & 7 \end{bmatrix}.$$

(f) Prove that,

$$\frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$$

is a unitary matrix.

(g) Prove that, if A is Hermitian matrix, then iA is a skew-Hermitian matrix.

4. Answer any ***three*** of the following questions : *3×5=15*

(a) Three urns A, B and C contain 6 Red and 4 Black balls, 2 Red and 6 Black balls and 1 Red and 8 Black balls respectively. An urn is chosen randomly and a ball is drawn from the urn. If the ball is Red, find the probability that the ball was drawn from urn A.

(b) If the variance of the Poisson distribution is 2, find the probabilities for $r = 1, 2, 3, 4$ from the recurrence relation of Poisson distribution. Also find $P(r \geq 4)$.

(c) Write *True* (T) or *False* (F) for the following :

If A and B represent two events and P(A) and P(B) are their probabilities of occurrence, then

(i) For events that are not mutually exclusive, the general additive rule is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

T/F

(ii) If A and B are exhaustive

$$P(A) + P(B) \neq 1$$ T/F

(iii) If A and B are independent

$$P(A/B) = P(A)$$ T/F

(iv) For mutually exclusive events, the additive rule is

$$P(A \text{ or } B) \neq P(A) + P(B)$$

T/F

(v) The multiplication rule for independent event is

$$P(A \text{ and } B) = P(A)\, P(B)$$

T/F

(d) Ten percent of screws produced in a certain factory turn out to be defective. Find the probability that in a sample of 10 screws chosen randomly, exactly two will be defective.

(e) A group of 200 students have the mean height of 154 cms. Another group of 300 students have the mean height of 152 cms. Can we say these are from the same population with standard deviation (SD) of 5 cms ?