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ET-102

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering)

Term-End Examination

December, 2015

00591

ET-102 : MATHEMATICS – III

Time : 3 hours *Maximum Marks : 70*

Note : *Question no. 1 is **compulsory**. Attempt any other **eight** questions from Q. no. 2 to Q. no. 15. Use of calculator is allowed.*

1. Fill in the blanks. All questions are ***compulsory***. *7×2=14*

   (a) The series $\sum \frac{1}{n}$ is ________ .

   (b) By D'Alembert's test, if Σu_n is a positive term series such that $\lim_{n \to \infty} \frac{u_{n+1}}{u_n} = l$, then the series converges if l ________ .

   (c) For Fourier series, if f(x) is an even function on $(-\pi, \pi)$, then $b_n =$ ________ .

   (d) The first order differential equation $M\,dx + N\,dy = 0$ is an exact differential equation, if there exists a function f(x, y) such that ________ .

(e) The solution of the differential equation $(D + 2)^3 y = 0$, where $D \to \frac{d}{dx}$, is ________.

(f) The Laplace Transform of $\{e^{3t} . \cos 4t\}$ is ________.

(g) The function $\frac{z^2+1}{(z-1)^2(z^2+4)}$ has three isolated singular points at $z =$ ________.

2. (a) Discuss the convergence or divergence for the following series : $3\frac{1}{2}$

$$\frac{1}{2} + \frac{1.3}{2.5} + \frac{1.3.5}{2.5.8} + \frac{1.3.5.7}{2.5.8.11} + \dots .$$

(b) Test the convergence of the series $\sum \left\{ \frac{(n^3+1)^{1/3} - n}{\log n} \right\}$. $3\frac{1}{2}$

3. Find the Fourier series for $f(x) = x \sin x$ in the interval $(-\pi, \pi)$. 7

4. Find the Fourier sine and Fourier cosine series of 7

$$f(x) = \begin{cases} x, & \text{when } 0 < x < \pi/2 \\ 0, & \text{when } \pi/2 < x < \pi . \end{cases}$$

5. (a) Determine the analytic function

$$f(z) = u(x, y) + i\, v(x, y),$$

if $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 2x + 1$. 4

(b) Find the Laurent's expansion of the function $f(z) = \frac{7z-2}{(z+1)\,z\,(z-2)}$ in the annulus $1 < |z+1| < 3$. 3

6. Prove that $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)}\,dx = \frac{\pi}{a+b}$,

where a, b > 0. 7

7. (a) Find the bilinear mapping that maps the points $z_1 = \infty$, $z_2 = i$ and $z_3 = 0$ into the points $w_1 = 0$, $w_2 = i$ and $w_3 = \infty$. 3

(b) Evaluate $\displaystyle\int_{0}^{2\pi} \frac{d\theta}{1 - 2a\cos\theta + a^2}$, $a^2 < 1$. 4

8. Use the method of variation of parameter to solve the differential equation

$(D^2 + 1)\,y = \tan x,\ 0 < x < \pi/2$, where $D \to \dfrac{d}{dx}$. 7

9. Find the series solution of

$$2x^2y'' - xy' + (1 + x)\,y = 0.$$ 7

10. Solve the P.D.E.

$$(D_x^2 - D_y^2 + D_x + 3D_y - 2)\,z = e^{x-y} - x^2y,$$

where $D_x = \dfrac{\partial}{\partial x}$ and $D_y = \dfrac{\partial}{\partial y}$. 7

11. Find the deflection u(x, t) of the vibrating string of length π, ends fixed and $c^2 = 1$ assuming zero initial velocity and K sin 2x as the initial deflection. 7

12. Find the inverse Laplace Transform

$$L^{-1}\left\{\frac{(3S-2)}{S^3(S^2+4)}\right\}.$$ 7

13. Using Laplace Transform, solve the differential equation

$$y'' + 7y' + 10y = 4e^{-3t},\ y(0) = 0,\ y'(0) = -1.$$ 7

14. Find the characteristic function, transfer function, frequency response function and characteristic roots of the equation

$$(D + 4D^{-1})\, x = f.$$ 7

15. A series circuit, in which both the charge and the current are initially zero, contains the elements $L = 1\ H$, $R = 1000\ \Omega$, $C = 6{\cdot}28 \times 10^{-6}\ F$. If the steady state current is produced in the circuit by an impressed alternating voltage $E = 100\ (\cos 100\ t)$, find by what fraction of a cycle does it lag or lead the voltage that produces it. 7