B.Tech. Civil (Construction Management) /
B.Tech. Civil (Water Resources Engineering) /
B.Tech. (Aerospace Engineering) /

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Term-End Examination

December, 2015

01771

ET-101 (B) : MATHEMATICS – II

(PROBABILITY AND STATISTICS)

Time : 3 hours   *Maximum Marks : 70*

Note : All questions are compulsory. Attempt any two
parts out of the three in each question. Each
question carries equal marks. Use of scientific
calculator is permitted.

1. (a) State and prove the addition rule for two
         events. Extend it to more than two
         events.

    (b) A card is drawn from a pack of 52 cards.
         Find the probability of getting a king or a
         heart or a red card.

    (c) Three technicians X, Y and Z service 20%,
         30% and 50% breakdowns, respectively,
         occurring on an automated production line.
         The technician X makes an incomplete
         repair 1 time in 20, Y makes an incomplete
         repair 1 time in 10, and Z makes an
         incomplete repair 1 time in 15. For the next
         breakdown a repair made was found to be
         incomplete. Find the probability that this
         repair was made by Z. $2 \times 7 = 14$

2. (a) What is a random variable ? What are its two types ? Explain with suitable examples. Define the distribution function of a random variable and state its important properties.

(b) A consignment of 10 similar PCs contains 4 defective PCs. If an institution makes a random purchase of 3 PCs from this consignment, find the probability distribution for the number of defective PCs purchased and compute the distribution function. Also draw its graph.

(c) Suppose that 5 out of 20 new buildings in a city violate the building code. What is the probability that a building inspector, who randomly selects 10 of the new buildings, will catch exactly 2 of the new buildings that violate the code ? $2\times7=14$

3. (a) Find the m.g.f. of a normal variate with mean 0 and variance 1. Show that all its odd order moments are zero.

(b) A machine automatically packs a chemical fertilizer in polythene packets. It is observed that 10% of the packets weigh less than 2·42 kg, while 15% of the packets weigh more than 2·50 kg. Assuming the weight of the packet is normally distributed, find the mean and the variance of the packet.

(c) A system contains a certain type of component whose lifetime X is exponentially distributed with mean of 5 years. If 8 such components are installed in different systems, then find the probability that at least 3 are still working at the end of 7 years. *2×7=14*

4. (a) Let X_1 and X_2 be two independent random variables each distributed uniformly in the interval [0, a], where a > 0 is a constant. Find the joint distribution of $X_1 + X_2$ and $X_1 - X_2$.

(b) The mean and variance of a population are μ and σ^2 respectively. Find the mean and variance of the sample mean $\overline{X}$ of a sample of size n.

(c) Two random samples gave the following results :

Sample	Size(n)	Sample mean ($\overline{x}$)	$\Sigma (x_i - \overline{x})^2$
I	10	15	90
II	12	14	108

Test whether the sample comes from the same normal population at 5% level of significance. *2×7=14*

5. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with mean μ and variance σ^2. Show that

$$T(\overline{X}) = \frac{2}{n^2} \sum_{i=1}^{n} i\, X_i \quad \text{is a consistent}$$

estimator of μ.

(b) The following are 10 measurements on some characteristic measured by the same instrument by two technicians A and B. Can we say that B is more consistent than A at 5% level of significance ?

A :	13	15	7	15	5	12	9	3	20	11
B :	12	7	2	8	6	9	5	7	6	8

(c) If 41 of 120 tyres of brand A failed to last 20,000 miles, while the corresponding figures for brand B and brand C of tyres are 27 of 80, and 22 of 100 respectively, test at 5% level of significance whether the three brands of tyres differ in quality. $2 \times 7 = 14$

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