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ET-101(A)

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) / BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI

Term-End Examination

December, 2015

ET-101(A) : MATHEMATICS – I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of calculator is permitted.*

1. Answer any ***five*** of the following : *5×4=20*

(a) Evaluate (any ***one***) :

(i) $\lim_{x \to 0} \frac{x^2}{1 - \cos 2x}$

(ii) $\lim_{x \to 2} \left[\frac{4}{x^2 - 4} - \frac{1}{x - 2} \right]$

(b) If $u = \cos\left(\frac{y}{x}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

(c) Find $\frac{dy}{dx}$, where $y = \tan^{-1}\left[\frac{\cos x - \sin x}{\cos x + \sin x}\right]$.

(d) If $\frac{dv}{dt} = -\frac{v^2}{100}$, and $v = 15$ when $t = 0$, find the value of t, when $v = 10$.

(e) Find the maximum and minimum values of $(1-x)^2 e^x$.

(f) Show that $V = \frac{A}{r} + B$ is the solution of the differential equation $\frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} = 0$.

(g) If $x^y = e^{x-y}$, find $\frac{dy}{dx}$.

(h) State Rolle's theorem and verify the theorem for $f(x) = x^2(x^2 - 1)$ in $0 \le x \le 1$.

2. Answer any ***four*** of the following : *4×4=16*

(a) Prove that $\int_0^{\pi/4} \log(1 + \tan\theta)\, d\theta = \frac{\pi}{8}\log 2$.

(b) Prove that $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx = \frac{\pi}{4}$.

(c) Evaluate $\int_1^{\infty} \frac{1}{1+x^2}\, dx$.

(d) Find the total area bounded by the curve $y = x(x-1)(x-2)$, and the axis.

(e) Find the area bounded by the parabolas

$$y^2 = 4ax \text{ and } x^2 = 4ay.$$

(f) Solve (any ***one***) :

(i) $(x + y + 1)\, dx - (2x + 2y + 1)\, dy = 0$.

(ii) $\frac{dy}{dx} + \frac{y}{x} = x^2$.

3. Answer any ***four*** of the following : *4×4=16*

(a) The position vectors of points P and Q are given by

$$\mathbf{r}_1 = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}, \mathbf{r}_2 = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}.$$

Determine **PQ** in terms of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$. Also compute the magnitude of **PQ**.

(b) Show that the vectors

$\mathbf{A} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{B} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, and

$\mathbf{C} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ form a right-angled triangle.

(c) Determine a unit vector perpendicular to the plane of

$\mathbf{A} = 2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, and $\mathbf{B} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$.

(d) A force given by $\mathbf{F} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, is applied at the point (1, –1, 2). Find the moment about the point (2, –1, 3).

(e) If $\mathbf{A} = x^2y\hat{\mathbf{i}} - 2xz\hat{\mathbf{j}} + 2yz\hat{\mathbf{k}}$, find curl (curl A).

(f) Show that

$\mathbf{A} = (6xy + z^3)\,\hat{\mathbf{i}} + (3x^2 - z)\,\hat{\mathbf{j}} + (3xz^2 - y)\,\hat{\mathbf{k}}$,

is irrotational. Find ϕ such that $A = \nabla \phi$.

4. Answer any ***six*** of the following : *6×3=18*

(a) Solve by Cramer's rule :

$2x - z = 1$

$2x + 4y - z = 1$

$x - 8y - 3z = -2$.

(b) If $A = \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}$,

show that $A^2 - 11A + 10I = 0$, where I and 0 are unit and zero, (3×3) matrices respectively.

(c) Find A^{-1}, if $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$.

(d) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$,

find the matrix X such that $A + X + I = 0$, where I and 0 are unit and zero (3×3) matrices respectively.

(e) Prove that

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca).$$

(f) Three matrices A, B and C are as follows :

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} -12 & 8 \\ 6 & -3 \end{bmatrix},$$

$$C = \begin{bmatrix} 2 & 6 \\ 3 & 8 \end{bmatrix}$$

Show that $A + C = 12\,B^{-1}$.

(g) If $\begin{bmatrix} x-z & -x-z \\ 7-t & 6+z \end{bmatrix} = \begin{bmatrix} 9-t & 5-t \\ t+5 & -x-y \end{bmatrix}$,

find the values of x, y, z and t.

(h) Determine the matrices A and B, where

$$A + 2B = \begin{bmatrix} 4 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} \text{ and}$$

$$2A - B = \begin{bmatrix} 3 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}.$$