# MCA (Revised) / BCA (Revised) 

Term-End Examination
December, 2015

## MCS-013 : DISCRETE MATHEMATICS

Time: 2 hours
Maximum Marks : 50
Note: Question number 1 is compulsory. Attempt any three questions from the rest.

1. (a) Write the truth value of the disjunction of "The earth is flat" and " $3+5=2$ ".
(b) If p and q are two propositions, then show that $\sim(p \vee q) \equiv \sim p \wedge \sim q$.
(c) Use Mathematical induction to prove that 4
$1+\frac{1}{4}+\frac{1}{9}+\ldots+\frac{1}{\mathrm{n}^{2}} \leq 2-\frac{1}{\mathrm{n}} \forall \mathrm{n} \in \mathrm{N}$.
(d) If $\mathbf{f}: \mathrm{R} \rightarrow \mathrm{R}$ is a function such that $f(x)=3 x+2$, prove that $f$ is one-one onto. Also, find the inverse of $f$.
(e) How many integers between 100 and 999 consist of distinct even digits?
(f) Show that the number of words of length n on an alphabet of $m$ letters is $\mathrm{m}^{\mathrm{n}}$.
2. (a) Prove that:

$$
\begin{equation*}
\frac{(n+1)}{(r+1)} C(n, r)=C(n+1, r+1) \tag{5}
\end{equation*}
$$

(b) Express the Boolean expression in three variables $\left(x+y^{\prime}+z^{\prime}\right)\left(x y+x^{\prime} z\right)$ in DNF.
3. (a) Two dice, one red and one white, are rolled. What is the probability that the white die turns up a smaller number than the red die?
(b) State and explain De Morgan's law for Boolean algebra. Also, explain duality principle with the help of an example.
(c) In how many distinct ways is it possible to seat eight persons at a round table?
4. (a) Use Mathematical induction to prove that

$$
1^{2}+2^{2}+3^{2}+\ldots+n^{2}=\frac{n(n+1)(2 n+1)}{6} \forall n \in N .
$$

(b) Find the Boolean expression $C$ for the following logic circuit :

(c) Prove the following equivalence:

$$
\sim \forall \mathrm{xP}(\mathrm{x}) \equiv \exists \mathrm{x} \sim \mathrm{P}(\mathrm{x})
$$

5. (a) Verify that $\mathrm{p} \wedge \mathrm{q} \wedge \sim \mathrm{p}$ is a contradiction and $p \rightarrow q \leftrightarrow-p \vee q$ is a tautology.
(b) Show that $\sqrt{2}$ is irrational.
