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BCS-012

BACHELOR OF COMPUTER APPLICATIONS (BCA) (Revised)

Term-End Examination

5814

December, 2015

BCS-012 : BASIC MATHEMATICS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number 1 is **compulsory**. Attempt any **three** questions from the rest.*

1. Attempt any ***eight*** parts from the following :

(a) Show that

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & 0 \end{vmatrix} = 0$$

where ω is a complex cube root of unity. *5*

(b) If $A = \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}$,

show that $A^2 - 4A + 5I_2 = 0$.

Also, find A^4. *5*

(c) Show that 133 divides $11^{n+2} + 12^{2n+1}$ for every natural number n. 5

(d) If p^{th} term of an A.P is q and q^{th} term of the A.P. is p, find its r^{th} term. 5

(e) If 1, ω, ω^2 are cube roots of unity, show that $(2-\omega)(2-\omega^2)(2-\omega^{19})(2-\omega^{23}) = 49$. 5

(f) If α, β are roots of $x^2 - 3ax + a^2 = 0$, find the value(s) of a if $\alpha^2 + \beta^2 = \frac{7}{4}$. 5

(g) If $y = ln\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$, find $\frac{dy}{dx}$. 5

(h) Evaluate :

$$\int x^2 \sqrt{5x-3}\, dx$$ 5

2. (a) If $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 3 \\ 3 & 0 & -1 \end{bmatrix}$, show that $A\,(\text{adj}.A) = |A|\, I_3$. 5

(b) If $A = \begin{bmatrix} 2 & -1 & 7 \\ 3 & 5 & 2 \\ 1 & 1 & 3 \end{bmatrix}$, show that A is row equivalent to I_3. 5

(c) If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$,

$B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, show that

$AB = 6\ I_3$. Use it to solve the system of linear equations $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$. *10*

3. (a) Find the sum of all the integers between 100 and 1000 that are divisible by 9. *5*

(b) Use De Moivre's theorem to find $\left(\sqrt{3} + i\right)^3$. *5*

(c) Solve the equation

$$x^3 - 13x^2 + 15x + 189 = 0,$$

given that one of the roots exceeds the other by 2. *5*

(d) Solve the inequality

$$\frac{2}{|x-1|} > 5$$

and graph its solution. *5*

4. (a) Determine the values of x for which $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is increasing and for which it is decreasing. *5*

(b) Find the points of local maxima and local minima of

$$f(x) = x^3 - 6x^2 + 9x + 2014, \; x \in \mathbf{R}.$$ 5

(c) Evaluate :

$$\int \frac{dx}{\left(e^x - 1\right)^2}$$ 5

(d) Using integration, find length of the curve $y = 3 - x$ from $(-1, 4)$ to $(3, 0)$. 5

5. (a) Show that

$$[\vec{a} - \vec{b} \quad \vec{b} - \vec{c} \quad \vec{c} - \vec{a}] = 0.$$ 5

(b) Show that the lines

$$\frac{x-5}{4} = \frac{y-7}{-4} = \frac{z-3}{-5} \text{ and } \frac{x-8}{4} = \frac{y-4}{-4} = \frac{z-5}{4}$$

intersect. 5

(c) A tailor needs at least 40 large buttons and 60 small buttons. In the market, buttons are available in two boxes or cards. A box contains 6 large and 2 small buttons and a card contains 2 large and 4 small buttons. If the cost of a box is ₹ 3 and cost of a card is ₹ 2, find how many boxes and cards should be purchased so as to minimize the expenditure. *10*
