M.Tech. IN ADVANCED INFORMATION TECHNOLOGY - NETWORKING AND TELECOMMUNICATION (MTECHTC)

Term-End Examination 00197

December, 2015

MINI-019 : STATISTICAL SIGNAL ANALYSIS

Time : 3 hours *Maximum Marks : 100*

Note :
- *(i) **Section - I** is **compulsory**. **Section - I** carries **30** marks.*
- *(ii) **Section - II** Answer **any five** questions. **Section - II** carries **70** marks.*
- *(iii) Assume suitable data wherever required.*
- *(iv) Draw suitable sketches wherever required.*
- *(v) Figures to the right indicate maximum marks.*

SECTION - I 10x3=30

1. (a) What are the elementary properties of probability ?

(b) Consider an experiment of drawing two cards at random from a bag containing four cards marked with the integers 1 through 4.

(i) Find the sample space S, of the experiment if the first card is replaced before the second is drawn.

(ii) Find the sample space S, of the experiment if the first card is not replaced.

(c) State the Baye's theorem of conditional probability.

(d) A lot contains 100 semiconductor chips, out of these 20 are defective. Two chips are selected randomly, without replacement, from the lot.

(i) What is the probability that the first one selected is defective ?

(ii) What is the probability that the second one selected is defective given that the first one was defective ?

(iii) What is the probability that both are defective ?

(e) Define discrete random variable and probability mass function.

(f) What is random process, describe it in detail ?

(g) Define and explain Markov process.

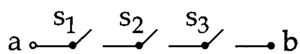
(h) Define the power spectral density. What are the various properties of it ?

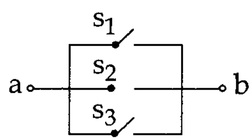
(i) List out four types of estimations.

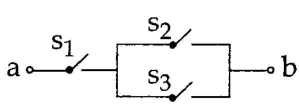
(j) Explain the M/M/1 queuing system.

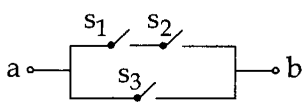
SECTION - II 5x14=70

2. Consider the switching networks shown in figure (a) - (d). Let A_1, A_2 and A_3, denote the events that the switches s_1, s_2 and s_3, are closed, respectively. A_{ab} denotes the event that there is a closed path between terminals a and b. Express A_{ab} in terms of A_1, A_2 and A_3 for each of the networks shown. **14**

(a)


(b)


(c)


(d)


3. Define probability density function. Let X be a continuous random variable X with pdf

$$f_x(x) = \begin{cases} kx & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Determine the value of k and sketch $f_x(x)$. **5**

(b) Find and sketch the corresponding cdf $F_x(x)$. **5**

(c) Find $P(1/4 < x \leq 2)$. **4**

4. Suppose the joint pmf of a bivariate r.v. (X, Y) is given by :

$$P_{xy}(x_i, y_j) = \begin{cases} \frac{1}{3} & (0, 1), (1, 0), (2, 1) \\ 0 & \text{otherwise} \end{cases}$$

(a) Are X and Y independent ? 7

(b) Are X and Y uncorrelated ? 7

5. (a) Differentiate between Markov chain and Markov process. 7

(b) Derive a two state Markov process and how it is used in digital communications ? 7

6. Consider a Markov chain with two states and transition probability matrix.

$$P = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$$

(a) Find the stationary distribution $\bar{p}$ of the chain. 4

(b) Find $Lim_{n \to \infty} P^n$. 5

(c) Find $Lim_{n \to \infty} P^n$ by first evaluating P^n. 5

7. Let $X(t)$ and $Y(t)$ be defined by

$X(t) = U \cos(w_0 t) + V \sin(w_0 t)$

$Y(t) = V \cos(w_0 t) - U \sin(w_0 t)$

where w_0, is constant and U and V are independent random variables both having zero mean and variance σ^2.

(a) Find the cross-correlation function of $X(t)$ and $Y(t)$. 7

(b) Find the cross power spectral density of $X(t)$ and $Y(t)$. 7

8. State and explain the following :

(a) Discrete random variable. 4

(b) Probability density function. 5

(c) Cumulative distribution function. 5