## M.Tech. IN ADVANCED INFORMATION Ұ TECHNOLOGY - SOFTWARE TECHNOLOGY (MTECHST) <br> Term-End Examination December, 2015 <br> MIN-004 : MATHEMATICAL FOUNDATION AND ALGORITHM DESIGN

Time : 3 hours
Note: (i)
(iv) Draw suitable sketches wherever required.
(v) Figures to the right indicate maximum marks.

## SECTION - I

1. Explain in detail binary search algorithm. The answer should contain the problem definition, the input, output and the procedure of algorithm. Write the definition of time complexity. Write time complexity of binary search algorithm.
2. Answer the following :
(a) One million trees grow in a forest. It is known that no tree has more six lakhs leaves. Show that at any moment there are two trees in the forest that have exactly same number of leaves.
(b) How many steps are needed to swap two variables? Does the answer change if we do not use the intermediate variable?
(c) Show that if any 26 people are selected then we may choose a subset of 4 people, so that all 4 were born on the same day of the week.

## SECTION - II

3. (a) (i) How many different arrangements are there of the letters of the word primes?
(ii) How many of those arrangements have $p$ as the first letter?
(iii) How many have p as the last letter or in any specified position?
(iv) How many will have $\mathrm{p}, \mathrm{r}$ and i together?
$2+2+2+2=8$
(b) Suppose $S$ is a set of $n+1$ integers. Prove 6
that there exist distinct $a, b \in S$ such that
$a-b$ is a multiple of $n$.
4. (a) Show that $(p \wedge q) \vee(\sim(q \wedge p))$ is tautology. 7
(b) Write a recursive algorithm to generate 7 Fibonacci number $F(n)$ for a given positive integer $n$ ?
5. (a) A door can be opened only with a security 7 code that consists of five buttons: $1,3,5,7$, 9. A code consists of pressing any one button, or any two, or any three, or any four, or all five. How many possible codes are there? (You are to press all the buttons at once, so the door doesn't matter).
(b) Find the inverse of $A=\left(\begin{array}{ccc}1 & -2 & 2 \\ 2 & -3 & 6 \\ 1 & 1 & 7\end{array}\right)$
6. (a) Give the definition and example of the 9 following :
(i) A complete graph
(ii) A pseudo graph
(iii) A multi graph
(b) Explain the divide and conquer algorithm 5 for problem solving.
7. (a) What is the degree of a vertex ? Draw a non-simple graph $G$ with degree sequence $(1,3,3,3,4,6,7)$.
(b) Find the minimal spanning tree of the graph 7 given below :

8. (a) Explain Bubble Soft Algorithm and its Time Complexity.

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5+2=7
$$

(b) Let $S=\{1,2,3\}$ and $R=\{(1,2),(2,2),(2,3)\}, \quad 7$ then find $\mathrm{R}^{+}$and $\mathrm{R}^{*}$.
9. (a) For each of the following, draw a Venn diagram and shade the region corresponding to the indicated set. $\quad 4+4=8$
(i) $\mathrm{A}-(\mathrm{B} \cap \mathrm{C})$
(ii) $(A-B) \cup(A-C)$
(b) Explain how mathematical techniques are required for design algorithms, with the help of example.

