# M.Sc. (MATHEMATICS WITH APPLICATIONS 

## IN COMPUTER SCIENCE)

M.Sc. (MACS)

00619 Term-End Examination
December, 2014

## MMTE-006 : CRYPTOGRAPHY

Time : 2 hours
Maximum Marks : 50
Note: Attempt any five out of 6 questions. Use of calculators is not allowed.

1. (a) Check whether the polynomial $f(x)=1+x+x^{2}+x^{5}+x^{6} \in Z_{2}[x]$ is irreducible with the help of an algorithm that checks the irreducibility of polynomials over finite fields.
(b) Given the sequence $111111000111111000 \ldots$, find the recurrence that generates it.
2. (a) Using Extended Euclidean Algorithm find $\operatorname{gcd}(f(x), g(x))$, where

$$
\begin{aligned}
& f(x)=1+x^{2}+x^{3}+x^{5}+x^{6}+x^{8} \\
& g(x)=1+x+x^{2}+x^{5}
\end{aligned}
$$

Also find polynomials $a(x), b(x)$ and $d(x)$ in $\mathbf{Z}_{2}[\mathrm{x}]$ such that $\mathrm{d}(\mathrm{x})=\mathrm{a}(\mathrm{x}) \mathrm{f}(\mathrm{x})+\mathrm{b}(\mathrm{x}) \mathrm{g}(\mathrm{x})$.
(b) What are the monoalphabetic and polyalphabetic substitution ciphers ? Compare them from the point of view of method for cryptanalysis.
3. (a) Use the keyword 'GATE' to encrypt the plaintext 'I STOOD FIRM TO FIGHT WAR' by Vigenère Cipher.

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(b) Assuming a block size of 64-bits and that each character is represented by 8 -bits, what will be the string you get by applying Merkle-Damgard strengthening to the string "tobeatornottobeat"?
(c) Explain the four basic steps in the round function of AES.
4. (a) In a long string of ciphertext, which was encrypted by means of an affine map on single-letter message units in the 26-letter alphabets, the most frequent letters are "K" and "H". Assuming that these ciphertext message units are encryption of "T" and "S", respectively, decrypt the ciphertext "DHKVVOUDEHKIKMIHK".
(b) In a field with $2^{8}$ elements and generator polynomial $p(x)=1+x+x^{3}+x^{4}+x^{8}$, find the product of the bytes $a=10011011$ and $b=00010100$ considered as elements of the field.
5. (a) Define Cryptographic Hash Function and Keyed Hash Function. What are the advantages of using Keyed Hash Function? 5
(b) Consider ElGamal Scheme with common prime $\mathrm{n}=19$ and primitive root $\alpha=10$. Bob has public key $\mathrm{y}_{\mathrm{B}}=3$ and Alice chooses a random integer $k=6$. What is the ciphertext of message $M=17$ ?
6. (a) Define pseudo prime, strong pseudo prime and Carmichael number. Prove that Carmichael number has at least three distinct prime factors.
(b) In a RSA public key cryptosystem the ciphertext sent to a user with public key $e=13, n=33$ is intercepted. If the intercepted ciphertext is $\mathrm{c}=8$, what is the plaintext M ?

