# M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) 

00402 M.Sc. (MACS)<br>Term-End Examination<br>December, 2014

## MMTE-002 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 2 hours
Maximum Marks : 50

Note: Do any five questions from questions no. 1 to 6. Use of calculators is not allowed.

1. (a) Find $9^{560}(\bmod 561)$ using modular exponentiation algorithm. 4
(b) Compute the Discrete Fourier Transform of (2, 2, -1, 1).
(c) Describe counting sort algorithm. Is counting sort stable? Justify your answer.
2. (a) Show that if all edges of a graph have distinct weights, then there exists a unique minimum weight spanning tree.5
(b) Consider the following algorithm :

Algorithm Find Min (A[1, 2, .., n])

$$
\begin{aligned}
& \min =\mathrm{A}[1] \\
& \text { for } \mathrm{i}=2 \text { to } \mathrm{n} \\
& \quad \text { do if } \mathrm{A}[\mathrm{i}]<\min \\
& \quad \text { then } \min =\mathrm{A}[\mathrm{i}]
\end{aligned}
$$

return min
Formulate a loop invariant and using it prove that the algorithm correctly finds minimum element.
3. (a) Illustrate worst case performance of quicksort, through an example.
(b) Show that, if $\mathrm{n} \geq 1$, then for any n -key $B$-tree $T$ of height $h$ and minimum degree $t \geq 2$ is given by $h \leq \log _{t} \frac{n+1}{2}$.
(c) Give the pseudocodes for MAKE-SET, UNION and LINK for finding disjoint-set forests.
4. (a) Simulate execution of Dijkstra's algorithm for the following example :

(b) Explain why the tree given in the figure below is not a binary search tree. Change the value of at most one key so that it becomes a binary search tree.


Also give the sequence of nodes examined if
(i) we apply the procedure Tree-Minimum.
(ii) we search the tree for the key 14.
(iii) we insert the key "9".
5. (a) Find an optimal parenthesization of a matrix-chain product whose sequence of dimension is $(5,10,3,12,5)$.
(b) What is the Huffman Code for the following set of frequencies :

| a | b | c | d | e | f |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 32 | 26 | 23 | 16 | 14 |

Show all the steps of the algorithm. Also, compute the number of bits required to encode the data.
6. (a) Assuming that all elements in a max-heap are distinct, where could possibly the smallest element reside in a max-heap ? Justify your answer.
(b) Define activity selection problem and describe a greedy algorithm for it.
(c) For the following network flow, draw the residual network :


Find the augmenting path $p$ and use it to augment the flow. Draw the flow network of the augmented flow.

