M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00902

December, 2014

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no. 1 is* ***compulsory****. Answer any* ***four*** *from the remaining six (2-7). Calculators or any other electronic devices are not allowed.*

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1. State, with justification or illustration, whether each of the following statements is *true* or *false*. *5×2=10*

(a) If G is a graph with K_3 as a sub-graph, then G is not a bipartite graph.

(b) If $\overline{G}$ is connected, then G is also connected.

(c) Every tree with two or more vertices is bipartite.

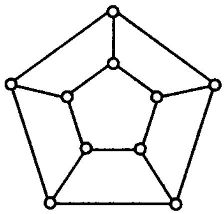
(d) The sequence

$$(5, 5, 5, 5, 5, 4, 4, 4, 4, 3, 3, 3, 2, 2, 1, 1)$$

is a graphic sequence.

(e) $K_{m, n}$ is Eulerian if m + n is even.

2. (a) Define isomorphisms of simple graphs. Check if the following graph G is isomorphic to the Petersen graph. 5



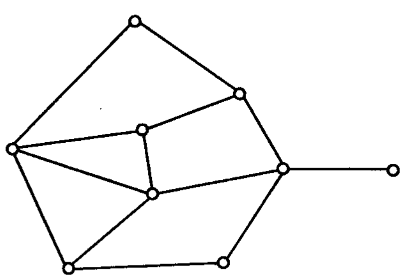
G

(b) Let v be a cut-vertex of a simple graph. Prove that $\overline{G}$ -V is connected. 5

3. (a) Prove that with n vertices, $n \geq 2$, in any simple graph there are at least two vertices of the same degree. 3

(b) Prove that a bipartite graph has a unique partition if and only if it is connected. 5

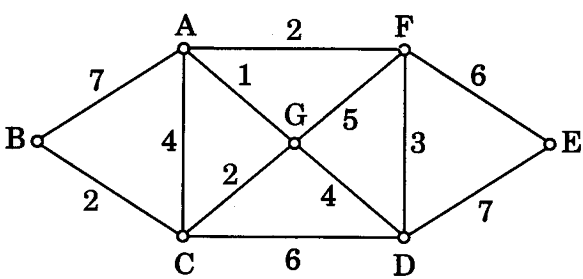
(c) Draw the dual graph of the following graph : 2



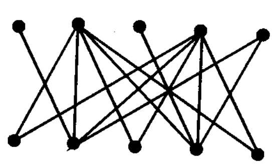
4. (a) Prove that every simple graph G has a bipartite subgraph with at least $\frac{e(G)}{2}$ edges. *3*

(b) If G is a simple graph with diameter at least three, then the diameter of its complement $\overline{G}$ is atmost three. *3*

(c) Use Kruskal's algorithm to find the minimum spanning tree of the following graph : *4*



5. (a) Find a maximum matching in the following graph. *5*



Prove that this is a maximum matching by exhibiting an optimal solution to the dual problem of finding minimum vertex cover.

(b) Define hypercube Q_n and determine its vertex connectivity. *5*

6. (a) Prove that the following statements are equivalent for a plane graph G : 4

   (i) G is bipartite.

   (ii) The dual graph G^* is Eulerian.

(b) State and prove Euler's formula for plane graphs. 3

(c) Let G be a graph with n vertices where chromatic polynomial is $k(k-1)^{n-1}$. Prove that G is a tree. 3

7. (a) Prove that any simple n-vertex graph with $\binom{n-1}{2} + 2$ edges is Hamiltonian if $n \geq 3$. 5

(b) Define an interval graph. Show that if G is an interval graph, then $\chi(G) = \omega(G)$. 5