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MMT-009

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00502

Term-End Examination

December, 2014

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Answer any* ***five*** *questions. Use of calculators is* ***not*** *allowed.*

1. (a) Explain the following terms giving example of each : 2

(i) Linear and Non-linear Models.

(ii) Discrete and Continuous Models.

(b) Fit a linear curve of regression to the following data : 3

x :	19	25	30	36	40	45	50
y :	76	77	79	80	82	83	85

2. Consider the discrete population growth model given by

$$N_{R+1} = N_R \exp\left[a\left(1 - \frac{N_R}{K}\right)\right]$$

for the population N_R where K is the carrying capacity and a is a positive parameter. Determine the non-negative steady states and discuss the stability of the model for $0 < a < 2$. Also find the first bifurcation value of the parameter. *5*

3. Do the stability analysis of one of the equilibrium solution of the following competing species system of equations with diffusion and advection :

$$\frac{\partial N_1}{\partial t} = a_1 N_1 - b_1 N_1 N_2 + D_1 \frac{\partial^2 N_1}{\partial x^2} - V_1 \frac{\partial N_1}{\partial x}$$

$$\frac{\partial N_2}{\partial t} = - d_1 N_2 + C_1 N_1 N_2 + D_1 \frac{\partial^2 N_2}{\partial x^2} - V_2 \frac{\partial N_2}{\partial x},$$

$$0 \leq x \leq 2$$

where V_1 and V_2 are constant advection velocities in x direction of the two populations with densities N_1 and N_2, respectively. a_1 is the growth rate, b_1 is the predation rate, d_1 is the death rate, C_1 is the conversion rate. D_1 and D_2 are diffusion constants. The initial and boundary conditions are

$$N_i\,(x, 0) = f_i(x) > 0, \;\; 0 \leq x \leq L, i = 1, 2$$

$$N_i = \overline{N_i} \text{ at } x = 0 \text{ and } x = L \;\forall\; t, i = 1, 2$$

where $\overline{N_i}$ are the equilibrium solutions of the given system of equations. Also, write the limitations of this model. *5*

4. (a) The owner of a readymade garments store sells two types of shirts – A and B. He makes a profit of ₹ 3 and ₹ 12 per shirt on A and B shirts, respectively. He has two tailors, X and Y, at his disposal, for stitching the shirts. Tailors X and Y can devote at the most 7 hours and 15 hours per day, respectively. Both these shirts are to be stitched by both the tailors. Tailors X and Y spend 2 hours and 5 hours, respectively in stitching one shirt A, and 4 hours and 3 hours, respectively in stitching one shirt B. Formulate this problem as LPP to maximize the daily profit. *2*

(b) The growth of a population is proportional to the population and is restricted by the lack of availability of resources like food, space, which can be modelled as proportional to the square of the population itself.

(i) Model this process.

(ii) Solve the resulting equation.

(iii) Show that in the long run the population approaches a limiting value. Give the interpretation of this value in the given context. *3*

5. (a) The control parameter of growth and decay of a tumour are, respectively 1000 and 500 per day. Also, the damaged cells migrate due to vascularization of blood at the rate of 200 cells per day. Find the ratio of the number of tumour cells after 50 days with the initial number of tumour cells. *2*

(b) Assume that the return distribution of two securities X and Y be as given below :

Possible rates of return of security		Associated probabilities
X	Y	$P_{Xj} = P_{Yj}$
0·11	0·18	0·42
0·17	0·16	0·15
0·10	0·11	0·30
0·19	0·09	0·13

Find ρ_{XY}. *3*

6. (a) Find the number of quantities required for estimating the expected return and standard deviation for 300 securities in Markowitz model and how many estimates are required for these securities while using single index Sharpe model. *2*

(b) A tax consulting firm has 4 service counters in its office for receiving people who have problems and complaints about their income, wealth and sales taxes. Arrivals average 80 persons in an 8-hour service day. Each tax advisor spends an irregular amount of time servicing the arrivals, which have been found to have an exponential distribution. The average service time is 20 minutes. Calculate the average number of customers in the system and average waiting time for a customer. *3*