No. of Printed Pages : 5

MMT-007

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00952

December, 2014

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no.* ***1*** *is* ***compulsory****. Do any* ***four*** *questions out of questions no.* ***2*** *to* ***7****. Use of calculators is* ***not*** *allowed. All computations may be kept to three decimal places.*

1. State, whether the following statements are *true* or *false*. Justify your answer with the help of short proof or a counter example. *5×2=10*

(a) In the Crank – Nicolson method, the partial differential equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ is replaced by the finite-difference equation

$$(1 + r)\, u_{i,\, j+1} = u_{i,\, j} + \frac{1}{2} r \,(u_{i-1,\, j+1} + u_{i+1,\, j} + u_{i+1,\, j+1} + u_{i-1,\, j} - 4u_{i,\, j}),$$

where $r = k/h^2$ and k and h are mesh spaces in dimension of t and x respectively.

(b) $\mathcal{L}\left[\int_0^t (t-z)^2 \sin z \, dz\right] = \frac{2}{s^2(s^2+1)}$,

where $\mathcal{L}$ denotes Laplace transform.

(c) If $P_n(x)$ is a Legendre's function, then

$$\int_{-1}^{+1} P_0(x) \, dx = 0$$

(d) The interval of absolute stability of the Runge – Kutta method

$y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$, $k_1 = h \, f(x_i, y_i)$, $k_2 = h \, f(x_i + h, y_i + k_1)$ is $-2 < \lambda h < 0$.

(e) The Lipschitz constant for the function $f(x, y) = x^2 |y|$, defined on $|x| \leq 1$, $|y| \leq 1$ is equal to 1.

2. (a) Find a series solution near $x = 0$ of the differential equation

$$9x(1-x)y'' - 12y' + 4y = 0.$$ 5

(b) Using Laplace transform, solve the differential equation $ty'' + y' + ty = 0$ with $y(0) = 2$, $y'(0) = 0$. 5

3. (a) If H_n is a Hermite polynomial of degree n, then show that

$$H_n'' = 4n\,(n-1)\,H_{n-2}.$$ 3

(b) Evaluate

$$\mathcal{L}^{-1}[\cot^{-1} s],$$

where $\mathcal{L}^{-1}$ denote inverse of Laplace transform. 2

(c) Solve the initial-value problem

$$y' = x^2 + \sqrt{y} + 1, \quad y(0) = 1$$

upto x = 0·4 using the predictor-corrector method

$$P : y_{n+1}^{(p)} = y_n + \frac{h}{2}(f_n - f_{n-1})$$

$$C : y_{n+1}^{(c)} = y_n + \frac{h}{12}\,[5\,f(x_{n+1}, y_{n+1}^{(p)}) + 8\,f_n - f_{n-1}]$$

with step length h = 0·2. Compute the starting value using Euler's method and perform two corrector iterations per step. 5

4. (a) Determine the appropriate Green's function by using the method of variation of parameters for the boundary value problem

$$\frac{d^2y}{dx^2} + y = e^{3x} \sin 2x \text{ with } y'(0) \geq 0,\; y(1) = 0.$$ 5

(b) Using Fourier integral transform, solve

$$\frac{\partial^4 z}{\partial x^4} + \frac{\partial^2 z}{\partial y^2} = 0 \quad (-\infty < x < \infty, y > 0)$$

satisfying the conditions (i) $z = f(x)$, $\frac{\partial z}{\partial y} = 0$ on $y = 0$, (ii) z and its partial derivatives tend to zero as $x \to \pm\infty$. 5

5. (a) Solve the boundary value problem $y'' - 5y' + 3y = 0$ with $2\,y(0) - y'(0) = 1$ and $y(1) + y'(1) = 2$, using second order finite difference method with $h = \frac{1}{2}$. 5

(b) Solve $\left(\frac{d^2y}{dx^2} - y\right)\left(\frac{d^2y}{dx^2} + y\right)^2 = 0.$ 2

(c) Find the Fourier transform of the function f(x) defined on $]-\infty, \infty[$, where

$$f(x) = \begin{cases} 1, & -l < x < 0 \\ 2, & 0 < x < l \\ 0 & \text{otherwise} \end{cases}$$ 3

6. (a) Find the interval of absolute stability for 3rd order Taylor Series Method to solve the initial boundary value problem

$$y' = \lambda y, \; y(x_0) = y_0.$$ 3

(b) Find the solution of $\nabla^2 u = 0$ in R subject to the boundary conditions $u(x, y) = x + y$ on Γ, where R is the square $0 \leq x \leq 1$, $0 \leq y \leq 1$ using the five point formula. Assume that step length is $h = \frac{1}{3}$ along the axis. 7

7. (a) For the explicit scheme

$$u_i^{n+1} = u_i^n + \lambda[u_{i+1}^n - 2u_i^n + u_{i-1}^n],$$

$\lambda = k/h^2 = \frac{1}{6}$, solve the parabolic equation $u_t = u_{xx}$, using Schmidt method. 3

(b) Find an approximate value of $y(1 \cdot 2)$ for initial value problem $y' = x^2 + y^2$, $y(1) = 2$ using the Adams – Moulton third order method

$$y_{i+1} = y_i + \frac{h}{12} [5 f_{i+1} + 8 f_i - f_{i-1}]$$

with $h = 0 \cdot 1$. Calculate the starting values using third order Taylor series method. 7