M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

00392

Term-End Examination

December, 2014

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

Note : *Question no.* ***1*** *is* ***compulsory****. Attempt any* ***three*** *questions from questions no.* ***2*** *to* ***5****. Use of calculators is* ***not*** *allowed.*

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1. State giving reasons whether the following statements are *true* or *false* : *5×2=10*

(a) $f(z) = |z|^2$ is nowhere differentiable.

(b) $f(z) = \sin z$ is a bounded function on **C**.

(c) The function $v(x, y) = e^x \sin y$, is an imaginary part of an analytic function.

(d) $\int_C \frac{dz}{z} = 0$ where $C : |z| = 1$.

(e) A linear fractional transformation which is not identity, has at most two fixed points in **C**.

2. (a) State necessary conditions for a function f(z) to be analytic in a domain D. Are these conditions sufficient ? Justify your answer. *3*

(b) Find the radius of convergence and the circle of convergence of the series *2*

$$\sum_{n=0}^{\infty} (-1)^{n+1} \frac{(z-1)^n}{\sqrt{n}}.$$

3. (a) Find the harmonic conjugate of the function $U(x, y) = e^{-y} \sin x$. *2*

(b) Evaluate $\int_C \frac{e^z \, dz}{z^{n+1}}$ where $C : |z| = 1$.

Hence deduce that

$$\int_0^{2\pi} e^{\cos\theta} \cdot \cos(n\theta - \sin\theta)\, d\theta = \frac{2\pi}{n!}.$$ *3*

4. (a) Expand $f(z) = \frac{z}{(z^2 + 1)}$ in a Laurent's series valid for the annular domain $0 < |z - i| < 2$. *2*

(b) Show that the transformation $w = \frac{1}{z}$ transforms the hyperbola $x^2 - y^2 = 1$ into a lemniscate $\rho^2 = \cos 2\phi$. *3*

5. Using contour integration, evaluate

$$\int_0^{\pi} \frac{d\theta}{a^2 + \sin^2 \theta}, \quad a > 0.$$ *5*
