No. of Printed Pages : 3

MMT-003

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00222

Term-End Examination

December, 2014

MMT-003 : ALGEBRA

Time : 2 hours *Maximum Marks : 50*

(Weightage : 70%)

Note : *Question no.* ***1*** *is* ***compulsory****. Answer any* ***four*** *questions from questions no.* ***2*** *to* ***6****. Use of calculators is* ***not*** *allowed.*

---

1. Which of the following statements are true and which are false ? Give reasons for your answers. *10*

(a) If every proper subgroup of a finite group G is cyclic, then G is cyclic.

(b) The action of $GL_n(\mathbf{R})$ on $\mathbf{R}^n$ by left multiplication is transitive.

(c) If a group is non-abelian, then it has an irreducible representation of degree greater than 1.

(d) If K/L is a Galois extension and F is a field such that $L \subseteq F \subseteq K$, then F/L is a Galois extension.

(e) The splitting field of $x^{15} - 1 \in \mathbf{Q}[x]$ has 15 elements.

2. (a) Prove that $SP_2(\mathbf{R}) = SL_2(\mathbf{R})$, but $SP_4(\mathbf{R}) \neq SL_4(\mathbf{R})$. *4*

(b) Determine the character table for the Alternating group A_4. *6*

3. (a) For $n \geq 4$, prove that the symmetric group S_n is not cyclic. Further, find the minimum number of elements required to generate S_n. *5*

(b) Determine the irreducible polynomial for $\alpha = \sqrt{2} + \sqrt{3}$ over each of the following fields (i) Q, (ii) $\mathbf{Q}(\sqrt{2})$. *5*

4. (a) Solve the set of congruences

$x \equiv 1 (\text{mod } 3),\ x \equiv 2 (\text{mod } 4),\ x \equiv 3 (\text{mod } 5).$ *4*

(b) Show that $PG(2, \mathbf{F}_q)$ is a projective plane. *6*

5. (a) Let P be a matrix in $SO_3(\mathbf{C})$. Prove that 1 is an eigenvalue of P. *4*

(b) Let G be a finite group with exactly two conjugacy classes. Prove that G must be cyclic of order 2. *4*

(c) Check if the ISBN number

978 – 0 – 143 – 06650 – 7

is a valid ISBN number. 2

6. (a) Let α be a complex root of the irreducible polynomial $x^3 - 3x + 4$. Find the inverse of $\alpha^2 + 1$ explicitly in the form

$$a + b\alpha + c\alpha^2, \; a, b, c \in \mathbf{Q}.$$ 3

(b) Let G be the group generated by x, y, z, with certain relations $\{r_i \mid i \in I\}$, where I is an indexing set. Suppose one of the relations has the form wx, where w is a word in y, z. Let r_i' be the relation obtained by substituting w^{-1} for x into r_i, and let G' be the group generated by y, z with relations $\{r_i' \mid i \in I\}$. Prove that G and G' are isomorphic. 5

(c) Give an example, with justification, of a regular language. 2